Journal of Asian Concrete Federation
Vol. 2, No. 1, pp. 81-94, June 2016
ISSN 2465-7964 / eISSN 2465-7972
http://dx.doi.org/10.18702/acf.2016.06.2.1.81

Estimation of concrete characteristic strength from limited data by bootstrap

Saha Dauji* and Kapilesh Bhargava

(Received: December 31, 2015; Accepted: May 19, 2016; Published online July 05, 2016)

Abstract: Compared to point estimate, interval estimate of compressive strength of concrete from limited test data would give a better confidence on the evaluated statistics, which can be obtained with Normal approximation theory. In this study, a non-parametric methodology based on bootstrap re-sampling for interval estimate was found to be more suitable. Optimal numbers of bootstrap samples and optimal number of data in each bootstrap sample in such analysis demand attention. Using cube test results, these aspects of bootstrap re-sampling as applied to interval estimate from concrete test data are investigated in this paper. It is recommended that for evaluation of mean or standard deviation of concrete compressive strength, optimum number of bootstrap samples should be between 1,000 and 2,000 with equal to or more than 25 data in each sample. The corresponding numbers for estimation of characteristic strength of concrete was advocated as 4,000 to 5,000 bootstrap samples, each of 30 or more data. Normal approximation theory might yield slightly higher estimate of characteristic strength, which could be detrimental in case of health evaluation or safety margin assessment of important existing structures.

Keywords: bootstrap, interval estimate, concrete test data analysis, non-parametric method, characteristic strength.

1. Introduction

At times, compressive strength of concrete is required to be evaluated from limited test data, which would be imprecise, and lack the estimate of the precision. This may be addressed by estimation of confidence interval of the statistics from Normal approximation theory as reported in literature.

It would be brought out in this paper that non-parametric interval estimates would be better suited for limited data sets. One simple non-parametric method is application of bootstrap re-sampling technique for interval estimate of the statistics of interest. For efficient application of the procedure, the effect of different numbers of bootstrap samples as well as the number of data in each bootstrap sample on such analysis demands due consideration. Although some applications of bootstrap for addressing issues related to concrete have been reported in literature, studies pertaining to its application in evaluation of statistics of compressive strength test data or estimate of characteristic strength of concrete are rare. In this paper, the various aspects of bootstrap sampling as applied to analysis of concrete test data are investigated. This is expected to provide the practicing engineers and analysts with general guidelines regarding selection of the sample size while applying bootstrap method for interval estimate of statistics and evaluation of characteristic strength from limited concrete test data. The study is based on the data of compressive strength of concrete obtained from cube test results, taken from a structure.

Bootstrap technique was first proposed by Efron for variance estimation of sample statistics based on observations (Efron and Tibshirani [1]). As compared to the classical statistical inferences based on normality conditions, bootstrap re-sampling is more generalized and versatile. Though bootstrap is computationally intensive, with today's computational resources it is not anymore a problem, and bootstrap may be efficiently applied for uncertainty analysis and confidence estimation for experimental data statistics. With the assumption that the observations are independent and come from the same distribution, bootstrap technique can be applied for interval estimate for mean, standard deviation, or any other statistic (Good and Hardin [2]). The interval indicates the precision of the corresponding point estimate.

Corresponding author Saha Dauji is a Scientific Officer, Bhabha Atomic Research Centre, Mumbai, India.
Kapilesh Bhargava is an Assistant General Manager, Bhabha Atomic Research Centre, Mumbai, India.

The bootstrap technique is to draw a certain number of samples, with replacements - randomly from the set of observations, with a probability assigned to each observation. This forms one bootstrap sample. From a certain number of such bootstrap samples, the interval estimate of the statistics of interest may be obtained. When equal probability is assigned to each observation, it is non-parametric bootstrap. In parametric bootstrap, the corresponding probability distribution parameters would be used for re-sampling process. In unbalanced bootstrap algorithm, the actual number of replications of individual sample points may not be equal to the number of bootstrap samples. The constrained algorithm, in which these two numbers are equal, is the balanced bootstrap technique. Further details of the method may be found in texts like Efron and Tibshirani [1] and Tung and Yen [3].

Babu and Bose [4] explored the confidence bounds obtained by non-parametric bootstrap for a wide class of statistics and compared them with those obtained by the Normal approximation theory. It was inferred from the results that the probability estimates of confidence intervals by bootstrap were unconditionally superior to the ones from Normal approximation theory.

Process capability index (C_{pp}) is an indicator for evaluating the capability of a process and estimation of confidence interval of the C_{pp} that is considered important for statistical inference on the process. Chou Chao-Yu et al. [5] studied the behavior of 95% bootstrap confidence intervals for estimating process incapability index (C_{pp}). Industrial processes are often idealized with non-normal distributions, and Burr distribution was used by the authors for the study. In general, larger sample size gave higher coverage percentage. While accounting the effect of non-normality, smaller skewness and kurtosis coefficients gave higher coverage percentage, with shorter confidence interval and smaller standard deviation of the bootstrap interval. From comparison of four bootstrap techniques, the standard bootstrap (SB), the percentile bootstrap (PB), the biased-corrected percentile bootstrap (BCPB), and the biased-corrected and accelerated (BCa), the coverage percentage of BCa interval was always found to be the best, followed by BCPB interval. It was also mentioned that the BCa interval produced the longer average interval length and the larger standard deviation of bootstrap interval length. Thus, bootstrap technique was found to be very efficient for evaluation of confidence intervals of process capability index.

In the event of mineral wastes reused in construction materials, leaching tests are performed for evaluation of the environmental impact of the reuse. Coutand et al. [6] applied bootstrap technique for quantitative evaluation of uncertainty of experimental data from leaching test on cement based materials as a function of the number of tests performed. In general, the interval was found to be wider for lower number of experimental observations. Large dispersion was observed in the experiments carried out and it was mentioned that analysis of leaching test results should be performed with caution. Confidence interval was estimated using bootstrap technique and standard procedure and it was observed that while the upper confidence limit was not significantly different in the two methods, the standard procedure overestimated the lower confidence limit.

In another study by Dauji et al. [7] the application of bootstrap for interval estimate of the mean and standard deviation of limited concrete test data was explored. The variations of the estimates with varying number of bootstrap samples from 200 to 5,000, each consisting of 20 samples, were examined and compared with those from the Normal approximation theory. The latter was found to overestimate the precision of the corresponding point estimates of statistics like mean and standard deviation of the test data. It was concluded that the non-parametric method of bootstrap is better suited for interval estimate of statistics than Normal approximation theory when dealing with limited test data of compressive strength of concrete. However, the effect of varying number of samples in each bootstrap sample was not studied. Further, the characteristic strength of concrete was not examined. These aspects would require attention for efficient application of the technique by practicing engineers.

In health and condition monitoring of important concrete structures, several properties of concrete like, the characteristic strength of concrete, the mean compressive strength of concrete, and its standard deviation play significant roles. Application of bootstrap technique for estimate of such properties from concrete test data is scarce in literature. As has been brought out in the foregoing discussion, bootstrap might be an efficient tool for interval estimate of these properties of concrete, especially when the available data is limited as in case of non-destructive test data of important structures. Thus, there exists scope of development of a set of guidelines for application of bootstrap methodology for interval estimate of concrete data statistics, and more importantly, its characteristic strength. This is the purpose of the present paper.

2. Data and methodology

Cube tests are required as a quality control measure for any concreting work executed and the results of the cube tests conducted during construction of a structure was available for the study. The structure was reinforced concrete (RC) framed type having approximate overall plan dimensions of 160 m X 175 m and consisted of six units separated by expansion joints. The different units had 2 to 6 stories of height 6 m each and were founded on raft 5 m below ground level. A few units had thick concrete walls above ground as well as partial basement with thick concrete external walls and internal partitions. The structure had a design concrete strength of 25 MPa, was designed according to IS codes of practice, and was constructed in late 1980s with the same grade of concrete as was used in design. This was an industrial structure having floor loadings in the order of 10 to 60 kN/m^2 and strict quality control was implemented during construction.

As a sample application of the bootstrap method for interval estimation of limited concrete test data statistics, 519 compressive strength data of cube tests from the structure were taken. Representative 40 samples were randomly selected from the complete set of test results to simulate the limited concrete test data. The unbalanced non-parametric bootstrap algorithm is presented in Fig. 1.

The non-parametric unbalanced bootstrap method was applied for interval estimation of the statistics of interest. For demonstration of the technique in strength estimation from concrete compression test results, the mean and standard deviation were chosen as statistics of interest. Dealing with sample size of 40, the number of data in each bootstrap sample was varied from 15 to 35 in steps of 2, while keeping the number of bootstrap samples like 200, 400, 600, 800, 1,000, 1,500, 2,000, 3,000, 4,000, and 5,000 and the variations studied. Interval estimate with Normal approximation theory was thereafter compared with those from bootstrap. Furthermore, sensitivity of the results to the choice of representative sample, different numbers of data in each bootstrap sample and different numbers of bootstrap samples were studied. Subsequently, the characteristic strength of concrete from bootstrap analysis of the compressive strength test data was evaluated and compared to that obtained by Normal approximation theory.

We propose a set of guidelines for application of the bootstrap method for interval estimate of the mean strength, the standard deviation, and the characteristic strength of concrete. This essentially consists of the optimal number of bootstrap samples and optimum number of data in each bootstrap sample to be used for interval estimate of each of the three variables and this is based on two considerations: computational efforts and accuracy of estimate. We plot the Box and Whisker plots that show the maximum, the minimum, the 1, 5, 25, 75, 95, and 99 percentiles for each case. The minimum number of samples/data beyond which the variations in the various percentiles stabilize indicate optimality and is recommended.

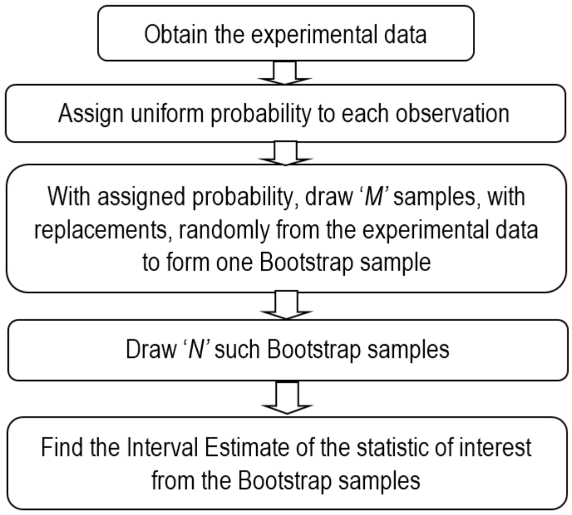


Fig. 1 – Flowchart for unbalanced non-parametric bootstrap technique

3. Results and discussion

3.1 Data characteristics

In this study, cube test data was taken from a structure that contained 519 records. Assigning equal probability to each of the 519 data, i.e. assuming uniform distribution, two different samples each of 40 data were randomly selected from the dataset. The descriptive statistics of population (519 data), and two representative samples, each consisting of 40 data, drawn as described above, are presented in Table 1.

Table 1 Descriptive statistics of population and samples

Statistic	Population	Sample 1	Sample 2
Max. (MPa)	39.90	39.80	39.90
Min. (MPa)	23.50	23.90	23.90
Mean (MPa)	35.62	35.50	35.48
Median (MPa)	36.50	36.55	36.55
Std. deviation (MPa)	3.57	3.67	3.71
C.O.V.	0.10	0.10	0.10
Skewness	-1.11	-1.15	-1.09
Kurtosis	3.86	3.98	3.86

It may be mentioned that the descriptive statistics of the two samples match well with that of the population, and can be taken as representative samples. From Table 1, it may be noted that the mean and the median of the data sets are quite close, and the minimum value is far from the mean and median, indicating a long tail towards the left. It may be also observed that the distribution of data is asymmetric towards the left, the negative skewness indicating that the frequency is more below the mean. The peakedness of the distribution is more than, but close to the Normal distribution (kurtosis = 3), as indicated by the kurtosis value. The coefficient of variation is around 10%, which indicates the variability of the observations.

For constructing the histograms, the data was grouped into intervals according to Eq. (1), where, *'I'* is the number of intervals and *'N'* is the number of records (Ranganathan [8]).

$$I = 1 + \log_{10} N \quad (1)$$

The histograms of the population and two representative samples are presented in Fig. 2. It may be observed that none of the histograms of the population or two of the bootstrap samples resembles normal distribution. It may be observed that around 37% of the data falls between 37 and 40 MPa while around 30% falls between 35 MPa and 37 MPa. As mentioned earlier, the design strength for the structure was 25 MPa. According to the Indian code of practice, adopting a conservative standard deviation of 4 MPa [9], the target strength would be 31.6 MPa [10, 11]. It may be noted that the standard deviation for mix design (4 MPa) is higher as compared to that obtained from the test results (3.57 MPa). Possibly owing both to the conservative mix design and to the strict quality control implemented in the construction of this important facility, the recorded results of the concrete cubes taken during construction display the observed localization around the higher values.

Normal and lognormal distributions were fit to the representative samples (40 data). Standard hypothesis tests like Chi-square and K-S tests were applied to the fitted distributions. Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) were also evaluated to judge suitability of the fitted distribution functions.

Normal distribution could not be rejected by Chi-square test at 10% and 1% level of significance, respectively, for sample 1 and Sample 2, and by Kolmogorov-Smirnov test at 10% level of significance for both. AIC and BIC for the fitted distributions were in agreement for both samples, and normal distribution was found to be more suitable by both metrics. The fitted distributions are

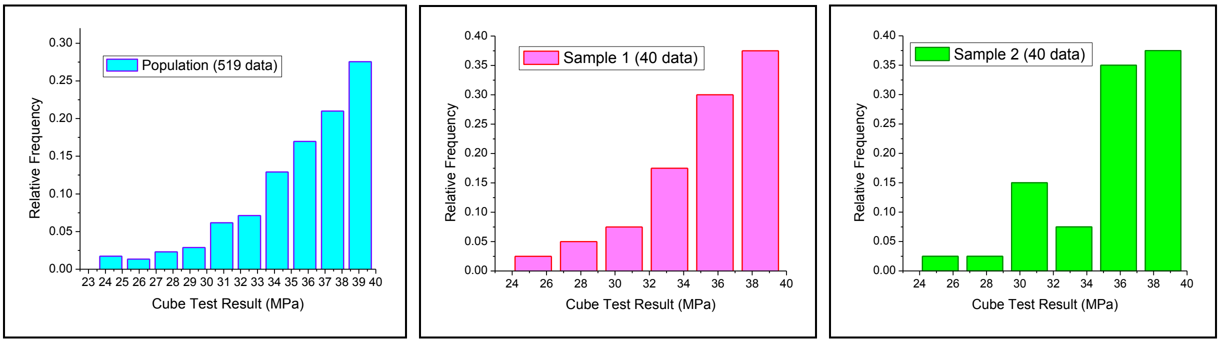
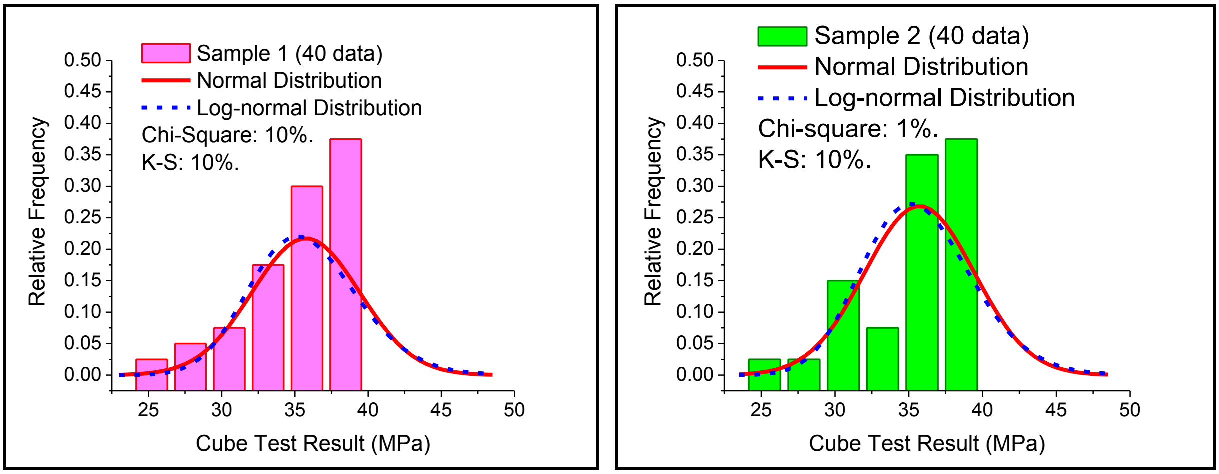


Fig. 2 – Histograms of population and representative samples



(a) Sample 1 (b) Sample 2

Fig. 3 – Fitted PDF for representative samples

presented in Fig. 3 for the two representative samples. However, keeping in mind the shapes of the histograms, it was felt that application of non-parametric method would be more appropriate for interval estimates.

3.2 Interval estimate by Normal approximation theory

The point estimate of the mean of concrete cube strength was 35.5 MPa for Sample 1 and 35.48 MPa for Sample 2, while the standard deviation was 3.67 MPa for Sample 1 and 3.71 MPa for Sample 2. Applying Normal approximation theory, the two-sided 99% confidence interval estimated for the mean of the Sample 1 was 33.93 MPa - 37.07 MPa and that of Sample 2 was 33.89 MPa - 37.07 MPa. Similarly, by Normal approximation theory, the two-sided 99% confidence interval estimated for standard deviation was 2.90 MPa - 5.53 MPa for Sample 1 and 2.93 MPa - 5.59 MPa for Sample 2.

3.3 Interval estimate by bootstrap: variations with number of bootstrap samples

The findings of interval estimate of the mean and standard deviation of the two samples by unbalanced non-parametric bootstrap is presented in Fig. 4 through Fig. 7. The maximum, the minimum, the 1, 5, 25, 75, 95, and 99 percentiles, and the mean obtained with different numbers of bootstrap samples for the samples from three runs are graphically depicted in the following Box-Whisker plots.

The different percentiles for mean of Sample 1 are plotted in Fig. 4. It may be observed that across the 3 runs, for 1,000 bootstrap samples and above, the mean and the median are almost same. Also, the 25 and 75 percentile falls between 35 MPa and 36 MPa, in all 3 runs. Furthermore, the range of 5 and 95 percentile is very close across the runs, roughly between 34 MPa and 36.8 MPa. There appears to be slight variations in the 1 and 99 percentile for any number of bootstrap samples, for all 3 runs. While for 1,000 bootstrap samples and above, the maximum hovers around 38 MPa, there exists substantial variation in the minimum value. This behavior is observed for all 3 runs.

In Figure 5, which depicts the different percentiles for mean of Sample 2, the near equality of mean and median, and range for 25 and 75 percentile between 35 MPa and 36 MPa are observed for all 3 runs, in the cases of 1,000 bootstrap samples and above. Furthermore, the range of 5 and 95 percentile is very close across the runs, roughly between 34 MPa and 36.8 MPa, as before. The variations of 1 and 99 percentiles for any number of bootstrap samples are noticed in all 3 runs, similar to Sample 1. However, in this case, the variation of the maximum value is slightly more than the earlier case, while the variation of the minimum value is similar.

From the foregoing discussion, it may be concluded that for the estimate of the mean from limited data, number of bootstrap samples should be more than 1,000. Considering the computational efforts, it is suggested to use 1,000-2,000 samples for interval estimate of mean from limited concrete test data.

The percentiles for standard deviation of Sample 1 are presented in Fig. 6. Across all runs, the mean and the median values are different for almost all number of bootstrap samples. The 25 and 75 percentile ranges from 3 MPa to 4 MPa, with slight variations. Beyond and including 1,000 samples, the 1, 5, 95, and 99 percentiles are similar for all 3 runs, and the values are around 2 MPa, 2.4 MPa, 4.7 MPa, and 5.2 MPa, respectively. The variations of the maximum and minimum are more compared to the other percentiles, and this can be observed in all runs.

From the percentiles for standard deviation plotted in Fig. 7 for Sample 2, it is observed that the mean is slightly higher than the median values for almost all number of bootstrap samples, across the runs. The 25 and 75 percentile ranges from 3 MPa to 4 MPa is maintained, with slight variations for 1,000 bootstrap samples and above. Beyond and including 1,000 samples, the 1, 5, 95, and 99 percentiles are similar for all 3 runs, and the values are around 2 MPa, 2.5 MPa, 4.8 MPa, and 5.2 MPa, respectively. This is almost similar to the values obtained for Sample 1. As before, the variations of the maximum and minimum are more compared to the other percentiles, and this can be observed in all runs.

It is suggested to adopt bootstrap samples equal to 1,000-2,000 for interval estimate of the standard deviation of limited concrete test data, in line with that for the mean. Further in this sub-section, the discussion is based on the estimates from 2,000 bootstrap samples.

The maximum, the minimum, and the spread obtained from the 3 bootstrap runs, each of 2,000 samples, and that obtained with Normal approximation theory are compared next. The values obtained for the same are presented in Table 2 for the mean and Table 3 for the standard deviation.

The upper and lower bounds for 99% confidence interval, obtained for different runs for the individual samples are in good agreement, indicating the robustness of the method. It can be observed that the upper bound for 99% confidence bound by Normal approximation theory and bootstrap technique obtained for the mean is quite close, with the former a little lower. Corresponding lower bound is

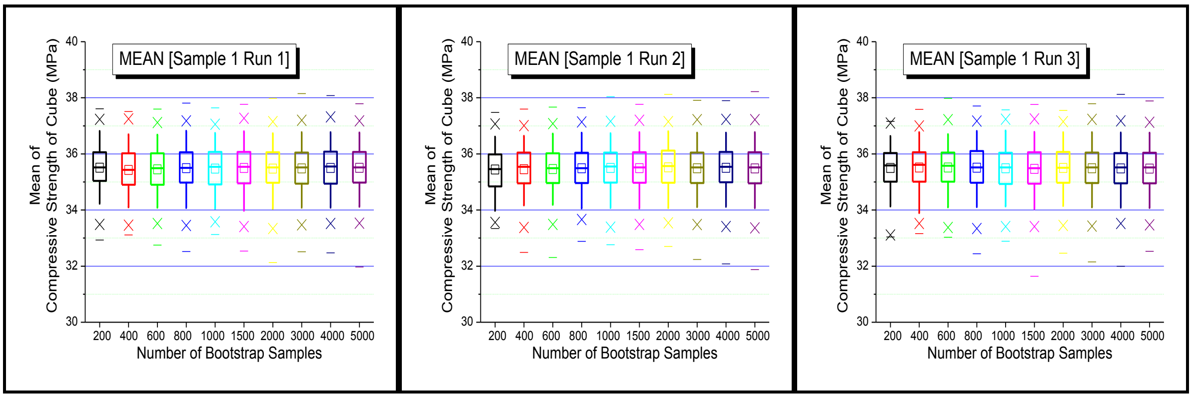


Fig. 4 – Box-Whisker plots for variations for 3 runs for Sample 1: mean

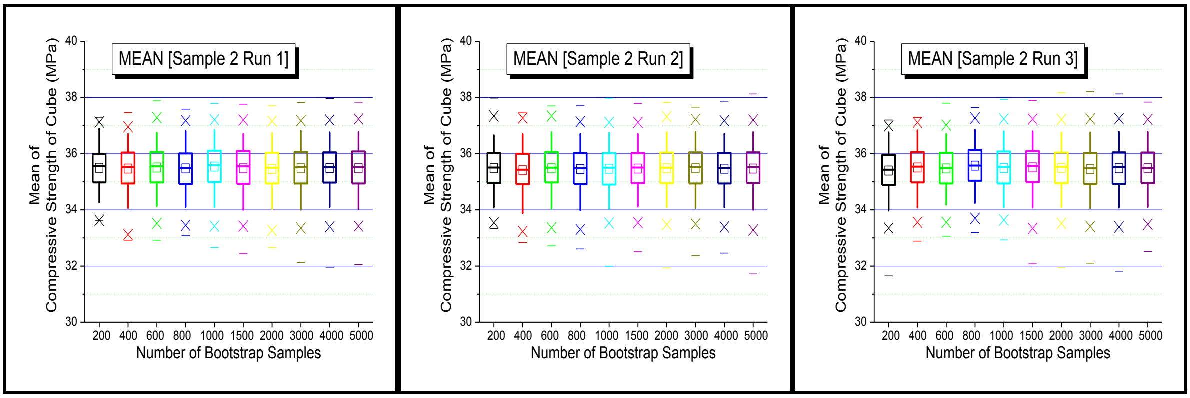


Fig. 5 – Box-Whisker plots for variations for 3 runs for Sample 2: mean

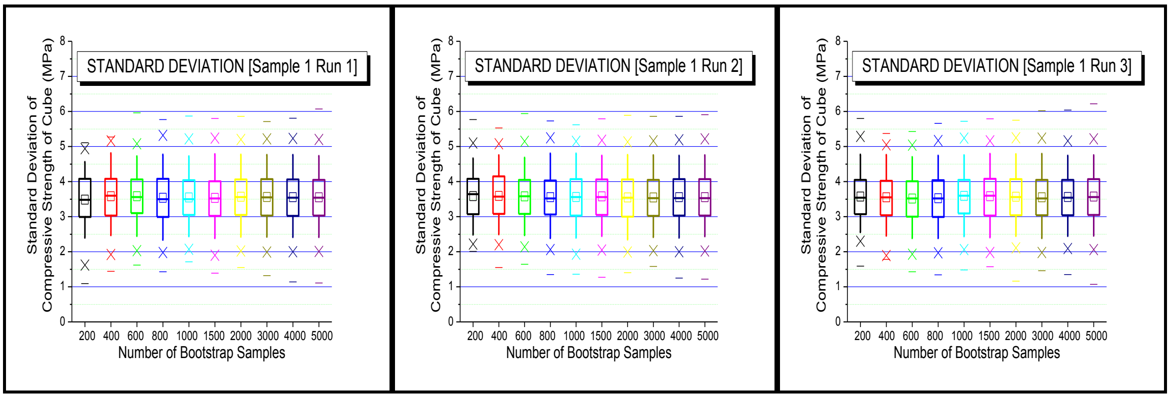


Fig. 6 – Box-Whisker plots for variations for 3 runs for Sample 1: standard deviation

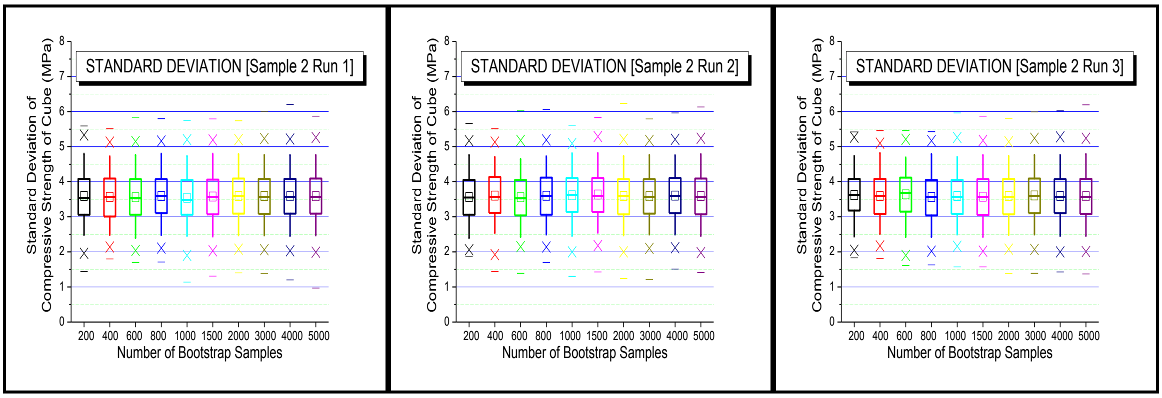


Fig. 7 – Box-Whisker plots for variations for 3 runs for Sample 2: standard deviation

Table 2 – Interval estimate for mean from 2,000 bootstrap samples and Normal approximation theory

Data	Method	Upper bound – 99% (MPa)	Lower bound – 99% (MPa)	Spread – 99% (MPa)
Sample 1	Bootstrap run 1	37.30	33.04	4.26
	Bootstrap run 2	37.35	33.39	3.96
	Bootstrap run 3	37.33	33.21	4.12
	Normal approximation theory	37.07	33.93	3.14
Sample 2	Bootstrap run 1	37.32	33.07	4.25
	Bootstrap run 2	37.47	33.23	4.24
	Bootstrap run 3	37.33	33.19	4.14
	Normal approximation theory	37.07	33.89	3.18

Table 3 – Interval estimate for standard deviation from 2,000 bootstrap samples and Normal approximation theory

Data	Method	Upper bound – 99% (MPa)	Lower bound – 99% (MPa)	Spread – 99% (MPa)
Sample 1	Bootstrap run 1	5.41	1.89	3.52
	Bootstrap run 2	5.32	1.83	3.49
	Bootstrap run 3	5.46	1.92	3.54
	Normal approximation theory	5.53	2.90	2.63
Sample 2	Bootstrap run 1	5.40	1.85	3.55
	Bootstrap run 2	5.40	1.88	3.52
	Bootstrap run 3	5.39	1.86	3.53
	Normal approximation theory	5.59	2.93	2.66

overestimated by the Normal approximation theory, yielding a narrower interval for mean, as compared to the bootstrap method. This behavior is observed in case of both the samples, and in similar lines as reported by Coutand et al. [6]. Thus, it may be inferred that the Normal approximation theory overestimates the precision of the point estimate of the mean, relative to bootstrap. Furthermore, as the lower bound of the mean value of compressive strength of concrete, for a particular confidence level would be of interest, Normal approximation theory would yield higher strength than obtained by applying non-parametric bootstrap.

The robustness of the bootstrap technique is again observed with the close agreement of upper and lower 99% confidence bounds of the standard deviation for the two samples. For both samples, it is observed that the upper bound as well as the lower bound is overestimated by Normal approximation theory as compared to the bootstrap, with the overestimation more in case of the lower bound. As reported by Coutand et al. [6], and also found in case of interval estimate of mean in the present study, the lower limit is affected in Normal approximation theory. Thus, like in case of mean, the precision of point estimate of the standard deviation of the samples is overestimated by Normal approximation theory, relative to the non-parametric bootstrap method applied here.

3.4 Interval estimate by bootstrap: variations with number of data in each bootstrap sample

The findings of interval estimate of the mean and standard deviation of the two samples by unbalanced non-parametric bootstrap is presented in Figs. 8 and 9. With a range of 1,000-2,000 bootstrap samples was established in the earlier subsection, the results presented in the present subsection focus on the runs with 1,500, 2,000, and 3,000 bootstrap samples. The maximum, the minimum, the 1, 5, 25, 75, 95, and 99 percentiles, and the mean obtained with different numbers of bootstrap samples for the samples from three runs are graphically depicted in the following Box-Whisker plots.

The percentiles for mean of Sample 1 and Sample 2 are presented in Fig. 8. It may be observed that for all cases, the median for mean is around 35.5 MPa. For 25 data in each bootstrap sample and above, the 25 and 75 percentiles are approximately 35 MPa and 36 MPa for any number of data in each bootstrap sample. Below 25 data in each bootstrap sample, even though the median is similar, the spread increases for all the percentiles, in general. In almost all the cases, the mean is slightly lower than the median. The variations of the maximum and minimum are more compared to the other percentiles, and this can be observed in all cases. This behavior is observed for both Sample 1 and Sample 2, for all three cases (1,500, 2,000, and 3,000 bootstrap samples) presented here.

The percentiles for standard deviation of Sample 1 and Sample 2 are presented in Fig. 9. It may be observed that, for and above 25 data in each bootstrap sample, the median for standard deviation is approximately 3.6 MPa and the 25 and 75 percentiles being around 3 MPa and 4 MPa, respectively. Below 25 data in each bootstrap sample, the spread is more for almost all the percentiles, though the median remains almost same. The variations of the maximum and minimum are more compared to the other percentiles, and this can be observed in all cases. This behavior is observed for both Sample 1 and Sample 2, for all three cases (1,500, 2,000, and 3,000 bootstrap samples) presented here.

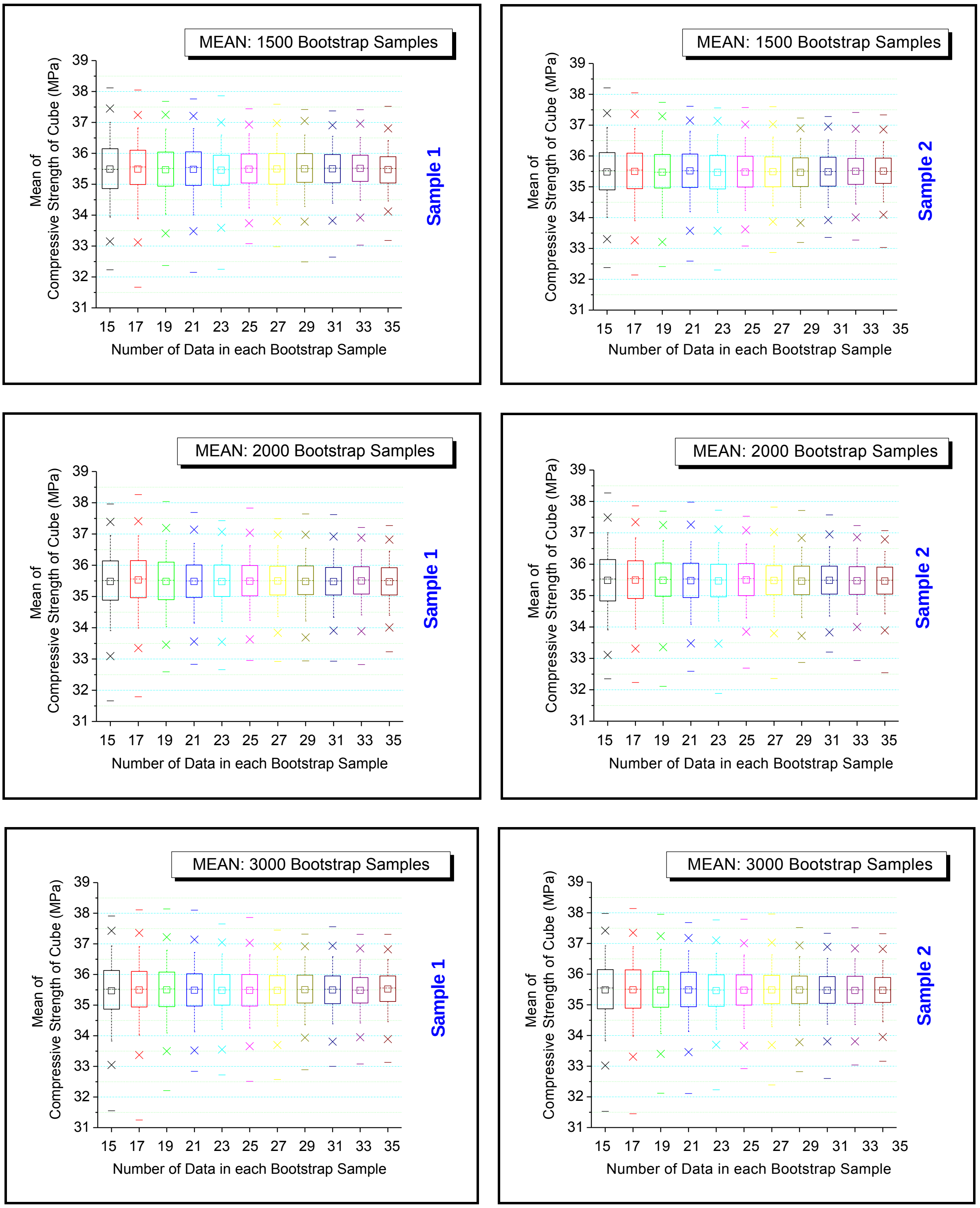


Fig. 8 – Box-Whisker plots for variations of mean for 1,500, 2,000, and 3,000 bootstrap samples (Sample 1 and Sample 2)

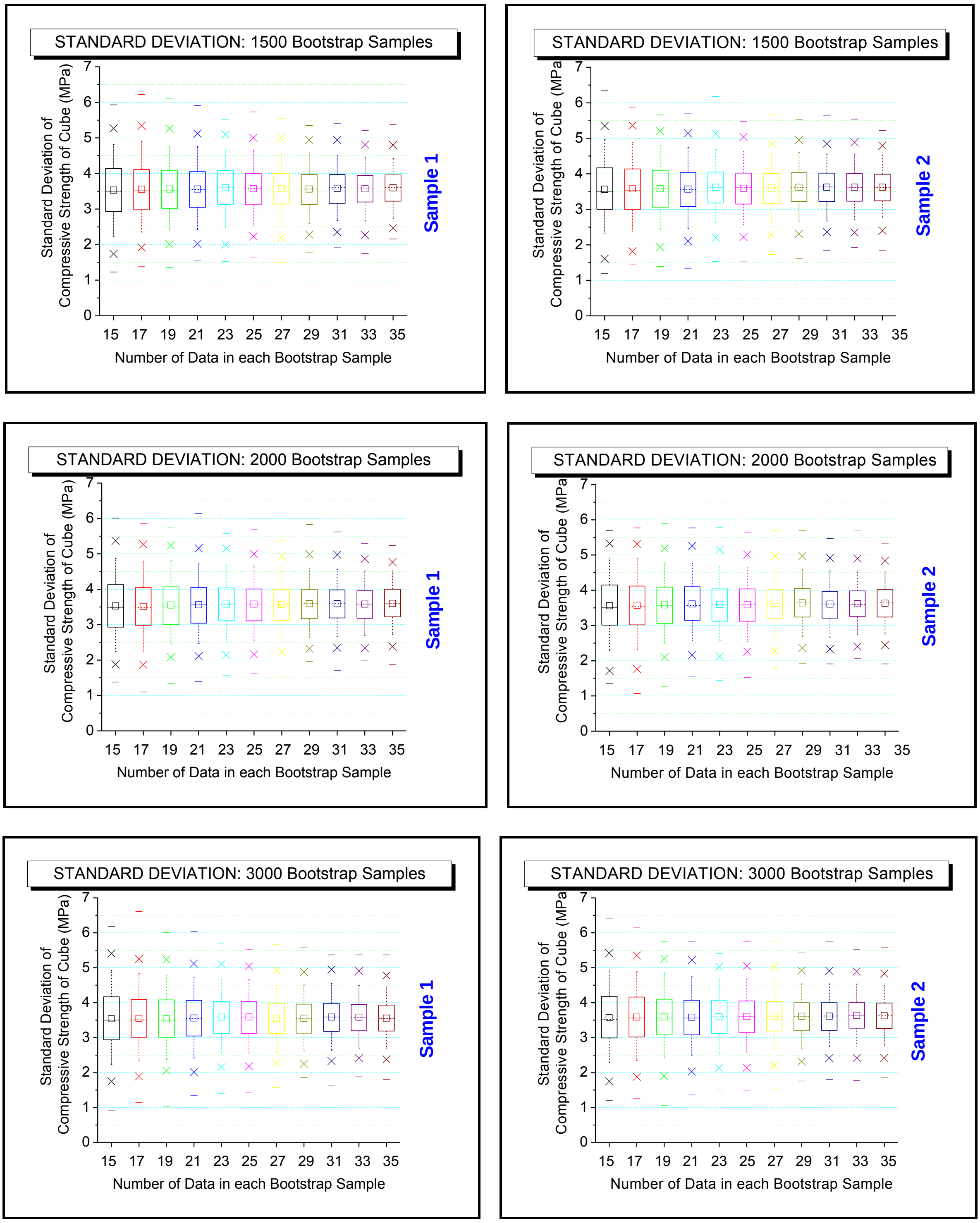


Fig. 9 – Box-Whisker plots for variations of standard deviation for 1,500, 2,000, and 3,000 bootstrap samples (Sample 1 and Sample 2)

Hence, it may be concluded that for the interval estimate of the mean and standard deviation from limited data, the number of data in each of the bootstrap samples should be equal to or more than 25. The lower and upper 99% bounds and the range obtained for mean and standard deviation of Sample 1 are presented in Table 4, Table 5, and Table 6 for 1,500, 2,000, and 3,000 bootstrap samples, respectively. It may be observed in Table 4, that for 1,500 bootstrap samples, with increasing number of data in each bootstrap sample in general, the lower bound is higher and the upper bound lower for both mean and standard deviation of the sample. This results in a narrower range with increasing data in each bootstrap sample. While for 15 data in each bootstrap sample, the ranges for mean and standard deviation are 4.85 MPa and 3.86 MPa, respectively, they come down to 3.0 MPa and 2.63 MPa for 35 data in each bootstrap sample.

Table 4 – 99% confidence interval for mean and standard deviation of Sample 1 (1,500 bootstrap samples)

Number of data in each bootstrap sample	Mean (MPa)			Standard deviation (MPa)		
	Lower	Upper	Range	Lower	Upper	Range
15	32.77	37.62	4.85	1.58	5.44	3.86
17	32.85	37.45	4.60	1.77	5.53	3.76
19	33.05	37.03	4.25	1.80	5.44	3.64
21	33.29	37.36	4.07	1.90	5.29	3.38
23	33.43	37.24	3.80	1.88	5.23	3.35
25	33.59	37.05	3.46	2.06	5.07	3.00
27	33.49	37.07	3.57	2.02	5.20	3.18
29	33.68	37.17	3.49	2.18	5.09	2.90
31	33.70	37.07	3.37	2.25	5.06	2.80
33	33.80	37.11	3.30	2.16	4.96	2.80
35	33.97	36.97	3.00	2.31	4.94	2.63
Normal approximation theory	33.93	37.07	3.14	2.90	5.53	2.63

Table 5 – 99% confidence interval for mean and standard deviation of Sample 1 (2,000 bootstrap samples)

Number of data in each bootstrap sample	Mean (MPa)			Standard deviation (MPa)		
	Lower	Upper	Spread	Lower	Upper	Spread
15	32.79	35.57	4.78	1.72	5.58	3.86
17	33.06	37.61	4.55	1.67	5.46	3.79
19	33.31	37.36	4.05	1.91	5.39	3.48
21	33.40	37.32	3.92	1.93	5.30	3.37
23	33.39	37.15	3.76	1.96	5.34	3.38
25	33.41	37.14	3.73	2.03	5.17	3.14
27	33.70	37.10	3.41	2.10	5.06	2.96
29	33.48	37.17	3.69	2.18	5.08	2.90
31	33.72	37.03	3.31	2.22	5.15	2.93
33	33.69	37.00	3.31	2.29	4.96	2.67
35	33.79	36.90	3.11	2.23	4.87	2.64
Normal approximation theory	33.93	37.07	3.14	2.90	5.53	2.63

Table 6 – 99% confidence interval for mean and standard deviation of Sample 1 (3,000 bootstrap samples)

Number of data in each bootstrap sample	Mean (MPa)			Standard deviation (MPa)		
	Lower	Upper	Spread	Lower	Upper	Spread
15	32.75	37.59	4.84	1.56	5.65	4.09
17	33.02	37.49	4.47	1.77	5.39	3.62
19	33.31	37.38	4.07	1.87	5.44	3.57
21	33.30	37.31	4.01	1.89	5.26	3.37
23	33.41	37.19	3.78	2.04	5.24	3.20
25	33.44	37.13	3.69	2.06	5.16	3.10
27	33.47	37.06	3.59	2.10	5.09	2.99
29	33.78	37.08	3.30	2.12	5.04	2.92
31	33.61	37.04	3.43	2.24	5.12	2.88
33	33.80	36.97	3.17	2.30	5.03	2.73
35	33.72	36.96	3.24	2.27	4.90	2.63
Normal approximation theory	33.93	37.07	3.14	2.90	5.53	2.63

It may be concluded that higher number of data in each bootstrap sample yields a narrower range for a particular confidence level, and indicates better accuracy of estimate of the statistics of interest. It may also be mentioned that compared to the Normal approximation theory, the upper and lower bounds as well as the range of the mean for 99% confidence level closely match the values obtained for number of data around 33-35 in each bootstrap sample. For standard deviation, although the range obtained with Normal approximation theory and that with number of data of 35 in each bootstrap sample are similar, the upper and lower bounds vary, and the bounds obtained by bootstrap are lower than those by Normal approximation theory. Overestimation of the lower bounds by Normal approximation theory has been earlier reported by Coutand et al. [6].

From Tables 5 and 6 for 2,000 and 3,000 bootstrap samples, respectively, it may be observed that behavior is similar to that observed for 1,500 bootstrap samples. In fact, for all the cases with 1,000 or more bootstrap samples, the same behavior could be observed. This indicates that the ranges for a particular confidence level obtained with bootstrap re-sampling are robust and not sensitive to the number of bootstrap samples in range 1,500 to 3,000.

Similar analysis of the lower and upper bounds as well as the ranges obtained with 99% confidence level for the mean and standard deviation of Sample 2 was performed for 1,500, 2,000, and 3,000 bootstrap samples. The values varied marginally from Sample 1, but behavior observed was similar. The results are not reproduced here for brevity. From the similarity of the values of bounds and ranges obtained for the two different representative samples from a population, it could be concluded that the results of bootstrap analysis would be insensitive to the choice of samples from population, and thus would be suitable while handling limited data for analysis.

3.5 Interval estimate by bootstrap: recommendations for analysis of limited concrete strength test data

In general, it may be inferred that the Normal approximation theory indicates higher precision of the point estimates as compared to the non-parametric unbalanced bootstrap method, as has been demonstrated by the samples from concrete compressive strength test data. The bootstrap method was found to be robust to the choice of initial sample data, the number of bootstrap samples, and the different bootstrap samples obtained in different runs. As discussed in section 3.3, it is suggested to adopt 1,000-2,000 bootstrap samples for optimum performance while evaluating interval estimate of the statistics with limited concrete test data. In section 3.4, it has been concluded that the number of data in each of those bootstrap samples should be equal to or more than 25.

3.6 Characteristic strength of concrete: estimate by bootstrap

According to the Indian code of practice for plain and reinforced concrete IS 456: 2000 (Fourth revision, reaffirmed in 2005 [9]), the characteristic compressive strength of concrete is defined as the strength of material below which not more than 5 percent of the test results are expected to fall. Thus, the lower 5% confidence level has special significance when dealing with concrete compressive strength data in India. It gives the characteristic strength of concrete as per the Indian standard.

As has been established in section 3.3 and section 3.4, the 5 percentile value of the mean strength of concrete was quite stable for the recommended bootstrap procedure. Hence, it is suggested that the lower 5 percentile value of mean strength of concrete compressive strength may be considered as its characteristic strength. The conditions for efficient estimation of the same are determined by studying the variation of this property with variations in number of bootstrap samples and number of data in each bootstrap sample. The results are presented in Table 7 and Fig. 10 for Sample 1, and Table 8 and Fig. 11 for Sample 2 respectively.

It may be observed that the range of characteristic strength obtained from the bootstrap analysis of Sample 1 is 33.81 MPa to 34.52 MPa (0.81 MPa) and that from Sample 2 is 33.82 MPa to 34.52 MPa (0.80 MPa), which is a very narrow band. It is also to be noted that the characteristic strength so obtained is insensitive to the sample chosen for analysis, and would be suitable for application in analysis of limited concrete strength data. With increasing number of data in each bootstrap sample, the characteristic strength is observed to increase in general. For 1,000 and 1,500 bootstrap samples, there are some fluctuations, which reduce for 2,000 and 3,000 bootstrap sample sets where the increase is almost monotonous. For 4,000 and 5,000 samples, the increase in characteristic strength with increasing number of data in each bootstrap sample is completely monotonous. Furthermore, it may be observed that, in general, the increase in characteristic strength saturates at around 30 data in each bootstrap sample, after which the increase is marginal. These are observed for both the samples analyzed.

Table 7 – Characteristic strength of concrete in Sample 1 (Lower 5 percentile of mean value)

Number of data in each bootstrap sample	No. of bootstrap samples					
	1,000	1,500	2,000	3,000	4,000	5,000
	Compressive strength (MPa)					
15	33.93	33.93	33.90	33.83	33.88	33.83
17	33.81	33.87	33.99	34.02	33.97	33.95
19	34.12	34.03	34.04	34.09	34.10	34.08
21	34.16	34.01	34.15	34.13	34.17	34.16
23	34.23	34.27	34.20	34.21	34.17	34.22
25	34.23	34.23	34.23	34.25	34.23	34.27
27	34.34	34.33	34.36	34.32	34.28	34.28
29	34.34	34.28	34.22	34.35	34.37	34.32
31	34.35	34.39	34.33	34.39	34.36	34.38
33	34.52	34.48	34.41	34.42	34.42	34.42
35	34.46	34.46	34.40	34.46	34.42	34.47
Normal approximation theory	34.52					

Table 8 – Characteristic strength of concrete in Sample 2 (lower 5 percentile of mean value)

Number of data in each bootstrap sample	No. of bootstrap samples					
	1,000	1,500	2,000	3,000	4,000	5,000
	Compressive strength in MPa					
15	33.92	34.01	33.91	33.82	33.90	33.83
17	33.88	33.90	33.94	33.98	33.97	33.96
19	34.06	33.99	34.12	34.06	34.07	33.98
21	34.08	34.17	34.09	34.13	34.14	34.12
23	34.10	34.17	34.18	34.21	34.14	34.19
25	33.83	34.23	34.28	34.23	34.24	34.23
27	34.29	34.38	34.33	34.29	34.32	34.29
29	34.31	34.32	34.30	34.33	34.33	34.32
31	34.36	34.32	34.34	34.37	34.38	34.32
33	34.29	34.44	34.40	34.36	34.38	34.38
35	34.52	34.51	34.42	34.44	34.42	34.45
Normal approximation theory	34.49					

3.7 Estimate of characteristic strength of concrete by bootstrap: recommendations for analysis of limited concrete strength test data

From the foregoing discussion, it is suggested to use around 30 data in each bootstrap sample and 4,000 to 5,000 bootstrap samples for evaluation of characteristic strength of concrete from limited test data. Upon comparison with the corresponding recommended values for interval estimate the mean and standard deviation by bootstrap, both the present values appear on the higher side. This could be due to the fact that uncertainty of estimates increases towards the tails of the distributions, higher number of samples are required to capture the same in the estimates. Another aspect which deserves mention is that the efficiency of bootstrap procedure is limited when it comes to estimation of the extremes, and more number of samples while estimating the characteristic strength would help.

Upon comparison with the characteristic strength obtained from Normal approximation theory, it is noticed that it falls in the higher end of the range obtained by bootstrap technique for both

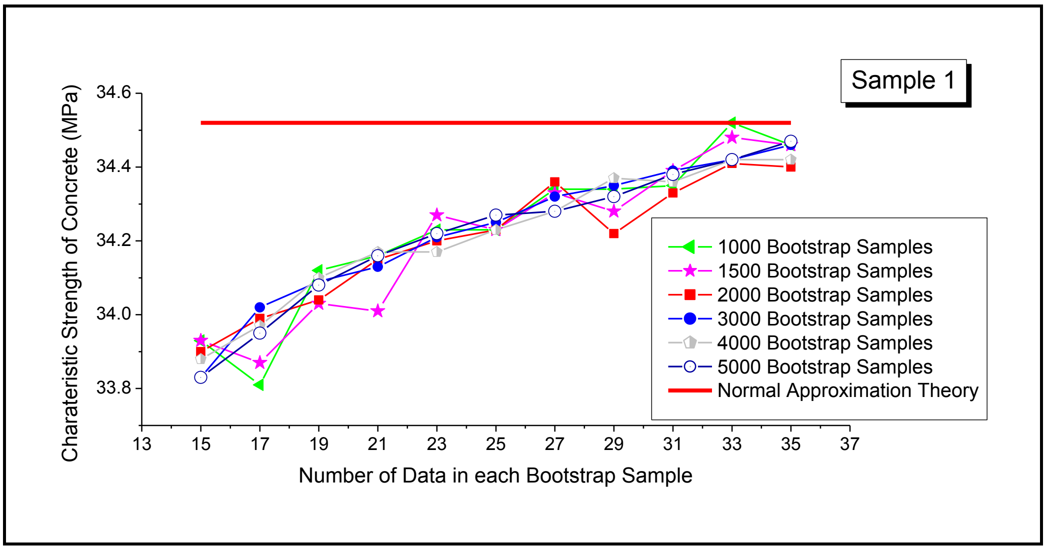


Fig. 10 – Variation of characteristic strength of concrete for Sample 1

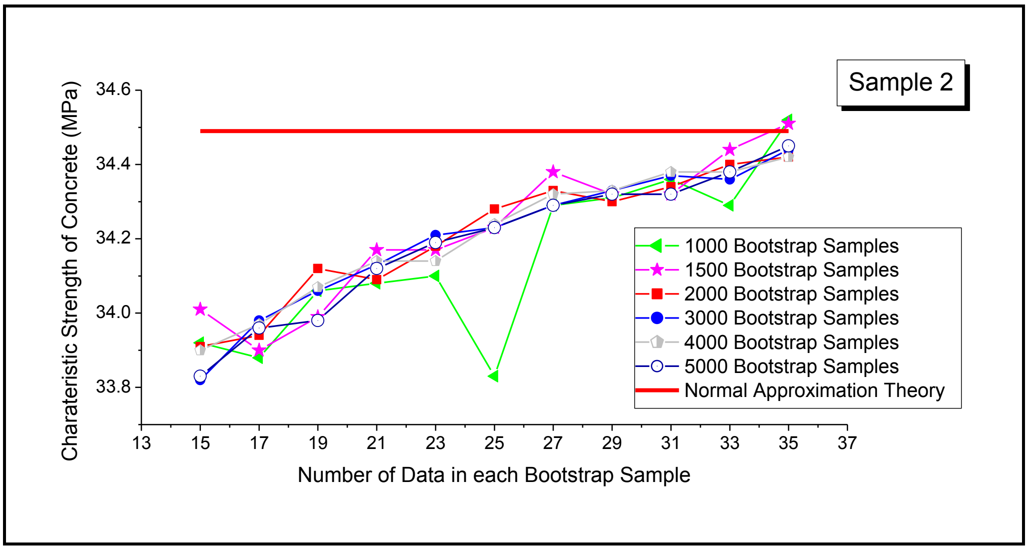


Fig. 11 – Variation of characteristic strength of concrete for Sample 2

samples. Thus, evaluation of characteristic strength by Normal approximation theory may yield slightly higher strength than that obtained by bootstrap, from limited test data. This would be especially significant when this characteristic strength goes into the health assessment or requalification of existing important concrete structures, where non-conservative values may give a false impression of safety.

4. Summary and conclusions

Interval estimate of descriptive statistics by re-sampling technique, namely, unbalanced non-parametric bootstrapping was demonstrated in this paper with limited concrete test data from a structure. In general, it was found that Normal approximation theory overestimated the precision of the corresponding point estimates of statistics like mean and standard deviation of the test data in the study on limited concrete test data. The unbalanced non-parametric bootstrap technique was observed to be robust to the choice of initial sample and different runs. From the results, it is inferred that the non-parametric method of bootstrap is better suited for interval estimate of statistics than Normal approximation theory when dealing with limited test data of compressive strength of concrete. It is suggested that optimum number of bootstrap samples may be between 1,000 and 2,000 for interval estimate of statistics from limited concrete test data. It is further advocated to use equal to or more than 25 data in each of those bootstrap samples.

From the later part of the study, it is concluded that evaluation of characteristic strength of concrete from the limited test data of compressive strength of cubes by application of Normal approximation theory would result in slightly higher value as compared to that obtained by bootstrap technique. The optimum number of bootstrap samples to be used for evaluation of characteristic strength is suggested to be between 4,000 and 5,000, with around 30 data in each of the bootstrap samples. Thus, for health evaluation and safety margin assessment where limited concrete cores are available to evaluate the insitu compressive strength of the concrete, Normal approximation theory would yield non-conservative characteristic strength of concrete. Non-parametric methodology like bootstrap would be better suited as it would give conservative value of the characteristic strength.

References

1. Efron, B. and Tibshirani, R. J. (1998) An Introduction to the Bootstrap, Chapman & Hall/CRC, Boca Raton.
2. Good, P. I. and Hardin, J. W. (2003) Common Errors in Statistics, John Wiley & Sons, New Jersey.
3. Tung, Y.-K. and Yen, B.-C. (2005) Hydrosystems Engineering Uncertainty Analysis, McGraw-Hill, USA.
4. Babu, G. J. and Bose, A. (1989) "Bootstrap Confidence Intervals," Statistics & Probability Letters, 7, pp. 151-160.
5. Chou, C.-Y.; Lin, Y.-C.; Chang, C.-L.; and Chen, C.-H. (2006) "On the Bootstrap Confidence Intervals of the Process Incapability Index C_{pp}," Reliability Engineering and System Safety, 91, pp. 452–459.
6. Coutand, M.; Cyr, M.; and Clastres, P. (2011) "Quantification of Uncertainty of Experimental Measurement in Leaching Test on Cement-Based Materials," Journal of Environmental Management, 92, pp. 2494-2503.
7. Dauji, S.; Bhargava, K.; and Basu, S. (2012) "Interval estimate of concrete test data statistics by bootstrap," Proceedings of International Congress on Computational Mechanics and Simulation (ICCMS 2012) IIT Hyderabad, India.
8. Ranganathan, R. (1999) Structural Reliability Analysis and Design, Jayco Publishing House, New Delhi, India.
9. IS 456: 2000, Plain and Reinforced Concrete - Code of Practice (Fourth Revision, reaffirmed 2005), Bureau of Indian Standard, India.
10. IS 10262: 2009, Recommended Guidelines for Concrete Mix Design, Bureau of Indian Standard, India.
11. SP 23: 1982, Handbook on Concrete Mixes (Based on Indian Standards), Bureau of Indian Standard, India.