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Technical Paper

Comparison of concrete strength from cube and core records by bootstrap

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Abstract: In this article, non-parametric method of bootstrap is employed for estimation of the concrete properties such as mean, standard deviation and characteristic strength from core test results obtained from anexisting structure and then compared to those obtained from the original cube results, as reported in literature. Estimates obtained from Normal approximation theory are reported for comparison purposes. The slightly higher estimate of mean for the cores could be attributed to the gain in strength of concrete over the years due to progressive hydration of cement. There is large variation in the standard deviation between the original cube results and the present core results, the latter being higher. The estimated characteristic strength from the cubes and the cores varied marginally in either method of estimation, indicating that the present concrete quality on the structure is as good as the time of casting more than 25 years back. This could be attributed to the good quality control during casting and periodic maintenance for preserving the quality as was performed for the structure under examination.

Keywords: bootstrap, concrete test data analysis, non-parametric method, characteristic strength, partially destructive tests.

1. Introduction

At times, compressive strength of concrete is required to be evaluated from limited test data, which would be imprecise, and lack the estimate of the precision. This may be addressed by estimation of confidence interval of the statistics from Normal approximation theory as reported in literature. Bartlett and MacGregor [1] discussed a simple procedure to evaluate the equivalent compressive strength of concrete from limited test results for assessment of the safety of an existing structure, which would be consistent with the statistical description of the initial design concrete strength. Bartlett [2] showed that confidence intervals estimates obtained by regression analysis underestimated the true model error and its accuracy could be improved by adopting a weighted regression analysis accounting for the non-uniform variances of the dependent and independent variables. In order to account for the scatter in the strength test data from concrete specimens, ACI 214.4R-03 [3] enumerates detailed provisions for arriving at the equivalent concrete strength by employing the tolerance factor approach as well as an alternate approach considering the level of confidence, which is expected from the estimate.

In such backdrop, nonparametric interval estimates could be better suited for limited data sets. One simple non-parametric method is application of bootstrap re-sampling technique for interval estimate of the statistics of interest. Some applications of bootstrap for addressing issues related to concrete have been reported in literature.

Bootstrap technique was first proposed by Efron for variance estimation of sample statistics based on observations [4]. As compared to the classical statistical inferences based on normality conditions, bootstrap re-sampling is more generalized and versatile. Though bootstrap is computationally intensive, with today's computational resources it is not anymore a problem, and bootstrap may be efficiently applied for uncertainty analysis and confidence estimation for experimental data statistics. With the assumption that the observations are independent and come from the same distribution, bootstrap technique can be applied for interval estimate for mean, standard deviation, or any other statistic [5]. The interval indicates the precision of the corresponding point estimate.

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The bootstrap technique is to draw a certain number of samples, with replacements - randomly from the set of observations, with a probability assigned to each observation. This dataset containing the desired number of samples forms one bootstrap sample. From a certain number of such bootstrap samples, the interval estimate of the statistics of interest may be obtained. When equal probability is assigned to each observation, it is non-parametric bootstrap. In parametric bootstrap, the corresponding probability distribution parameters would be used for re-sampling process. In unbalanced bootstrap algorithm, the actual number of replications of individual sample points may not be equal to the number of bootstrap samples. The constrained algorithm, in which these two numbers are equal, is the balanced bootstrap technique. Further details of the method may be found in texts like Efron and Tibshirani [4] and Tung and Yen [6].

Babu and Bose [7] explored the confidence bounds obtained by nonparametric bootstrap for a wide class of statistics and compared them with those obtained by the Normal approximation theory and inferred that the probability estimates of confidence intervals by bootstrap were unconditionally superior to the ones from Normal approximation theory.

Chou Chao-Yu et al. [8] studied the behavior of 95% bootstrap confidence intervals for estimating process capability index (C_{pp}), an important indicator for evaluating the capability of a process. They employed Burr distribution for the same. From comparison of four bootstrap techniques, namely, the standard bootstrap (SB), the percentile bootstrap (PB), the biased-corrected percentile bootstrap (BCPB), and the biased-corrected and accelerated (BCa), the coverage percentage of BCa interval was always found to be the best, followed by BCPB interval. Bootstrap technique was found to be very efficient for evaluation of confidence intervals of process capability index.

Coutand et al. [9] applied bootstrap technique for quantitative evaluation of uncertainty of experimental data. Data for their study was from leaching test on cement-based materials. The uncertainty was estimated as a function of the number of tests performed. In general, the interval was found to be wider for lower number of experimental observations. Confidence interval was estimated using bootstrap technique and standard procedure and it was observed that while the upper confidence limit was not significantly different in the two methods, the standard procedure overestimated the lower confidence limit.

In another study by Dauji et al. [10] the application of bootstrap for interval estimate of the mean and standard deviation of limited concrete test data was explored and compared to Normal approximation theory. The latter was found to overestimate the precision of the corresponding point estimates of statistics such as mean and standard deviation of the test data. It was concluded that the non-parametric method of bootstrap is better suited for interval estimate of statistics than Normal approximation theory when dealing with limited test data of compressive strength of concrete. However, the effect of varying number of samples in each bootstrap sample was not studied. Further, the characteristic strength of concrete was not examined.

The authors had later explored the estimation of concrete properties such as mean, standard deviation, and characteristic strength of concrete from limited data obtained from cube strength results by bootstrap [11], addressing the limitations as mentioned above. It was concluded that for mean and standard deviation, optimal number of bootstrap samples would be between 1,000 and 2,000 with more than 25 data in each sample. For characteristic strength, the corresponding numbers would be 4,000 to 5,000 with each sample containing 30 or more data. Normal approximation theory yielded slightly higher estimates, which could be detrimental in case of health evaluation of important structures.

In health and condition monitoring of important concrete structures, several properties of concrete such as the characteristic strength of concrete, the mean compressive strength of concrete, and its standard deviation play significant roles. For health evaluation of important structures, the in situ concrete strength would be estimated from non-destructive and partially destructive testing. The number of partially destructive test results, such as core strength results, would be limited in number in order to restrict additional distress to the structure due to testing. Here, application of bootstrap would be particularly effective for evaluation of the concrete characteristic strength, which would form an input for the re-analysis of the structure and health assessment. In case the cube test results obtained at the time of construction of the structure is also available, comparison of the characteristic strength obtained from the cube strength results and the concrete core strength results would be interesting.

It has been reported in literature [12] that contrary to the argument that the core strength taken after many months of the casting would have strength higher than that of 28 day cube strength, the core results have been observed to be less than the 28 days cube strength in most cases of normal strength concrete. Even for high strength concrete the core results up to one year were less than that of 28 day cube strength. The possible reasons for this phenomenon would include location of the core in the structure, position of the core with respect to the lift height, poor curing, presence of tensile cracks, presence of trapped bleed water, and the direction of the cores [12].

In this article, the concrete properties of an existing structure are evaluated from the core test results by applying bootstrap procedure. The concrete properties evaluated included mean, standard deviation and characteristic strength. Estimates obtained from Normal approximation theory are reported for comparison purposes. The characteristic strength as defined in the Indian standard [13] as the strength below which not more than 5% of results are expected to fall, or in other words, the lower 5 percentile strength. These properties were earlier evaluated by the authors [11] from the cube strength results from the same structure by application of bootstrap and these are now compared with those obtained from the core results. The variation in compressive strength of the structure as obtained from the cube test results at the time of casting and the present day core test results would be studied.

2. Data and methodology

An important facility was required to be seismically re-qualified due to revision in ground motion parameters and loads. The structure was reinforced concrete (RC) framed type having approximate overall plan dimensions of 160 m X 175 m and consisted of six units separated by expansion joints. The different units had 2 to 6 storeys of height 6 m each and were founded on raft 5 m below ground level. Few units had thick concrete walls above ground as well as partial basement with thick concrete external walls and internal partitions. The structure had a design concrete strength of 25 MPa, was designed according to then-prevailing IS code of practice [13] and was constructed in late 1980s with the same grade of concrete as was used in design. This was an industrial structure having floor loadings in the order of 10 to 60 kN / m^2 and the design of the concrete mix was according to the then-prevailing codes of practice [14, 15]. Being an important facility, strict quality control was implemented during construction. During the service life, periodic maintenance was performed to ensure continued health of the structure.

A comprehensive non-destructive testing exercise was conducted for estimation of the recent condition of the concrete elements like slabs, beams, columns, beam-column junction, walls, corbels, etc. with regard to strength, carbonation, corrosion, etc. The results of visual inspection indicated that the present concrete quality was apparently good. Limited number of partially destructive tests such as core tests was performed at carefully identified locations over the entire structure. The cores were 69 mm in diameter and, for testing, length of samples was kept at twice the diameter. A total of 60 core test records was available for the structure.

Care was taken to exclude presence of reinforcement in the core samples by employing detailed profoscope survey. The direction of cores in the columns, and beams was horizontal, which was perpendicular to the direction of placement and compaction. For slabs, the direction of cores was vertical, same as the direction of placement and compaction. Due to the continued operation requirement and functional limitation of the facility, the number of cores was very limited. Thus, the effect due to the core direction or that due to diameter could not be accounted for in the present study. Strength loss due to the variation of the moisture content between the test cores and the in situ concrete was ignored, as conservative estimate of the strength was intended. The length-to-diameter ratio in the present case was 2.0 and hence no correction was required on this account.

For reducing the core test strength to the equivalent concrete strength, various empirical factors reported in literature [12] vary from 0.8 to 0.89.As in this case a conservative concrete strength estimate was desired for the purpose of seismic re-evaluation of the structure for the present day loads, a conservative conversion factor of 0.8 was used to obtain the equivalent cube strength from core results.

Table 1 – Descriptive statistics of the cube and core test results

Statistic	Original cube strength (MPa)	Total core test 60 nos. (MPa)	Core Set 1, 30 nos. (MPa)	Core Set 2, 30 nos. (MPa)
Max.	39.90	54.10	54.10	53.60
Min.	23.50	21.00	23.80	21.00
Mean	35.62	39.59	39.09	40.09
Median	36.50	38.55	38.45	38.70
S.D.	3.57	7.27	7.18	7.45
C.O.V.	0.10	0.18	0.18	0.19

NOTE: S.D. – standard deviation; C.O.V. – coefficient of variation

The equivalent cube strength of concrete obtained from the core test results was available for the study and the same has been referred as the 'core strength' results throughout this article. The cube strength results obtained during casting of the structure have been referred to as 'original cube strength'.

The 60 numbers of core test records were split randomly into 2 sets, designated as Core Set 1 and Core Set 2 subsequently. The descriptive statistics of the original cube results, the total core results, and the two random sets generated from the core results are given in Table 1. The statistics of the random sets are similar to each other as well as the total core test results, which indicates the representativeness of the two core sets. Further, it can be observed that while the range of the compressive strength almost doubled from 16 MPa in original cube results to 33 MPa in the core results, the mean and median for the core results are within 10% and 5% of the corresponding estimates from the original cube results. The standard deviation and the coefficient of variation both almost doubled in the core results indicating more spread in the core data as compared to the cube strength accompanied with a positive shift of the central value. The higher spread may be attributed to various factors like the locations of the core in the structure, positions of the core with respect to the lift height, variation in curing, presence of tensile cracks, presence of trapped bleed water, and the directions of the core [12].

The histograms of the original cube results, the core results, and the two core sets are presented in Fig. 1. It is observed that the shape of the histogram for original cube and the core tests are quite different. The histogram of the original cube results is asymmetric with a lot of data towards the higher side. Concrete strength test data are generally assumed to follow normal distribution. The concentration of the data towards the higher end, which happens to be around the target strength of the mix design may be attributed to a well designed mix, consistency of the materials used in casting, and excellent quality control during construction. In case of the core results, there is slight asymmetry with a tendency towards the right, but the data is more evenly distributed than the original cube results. There is good similarity between the histograms of the total core results and the two random sets: Core Set 1 and Core Set 2.

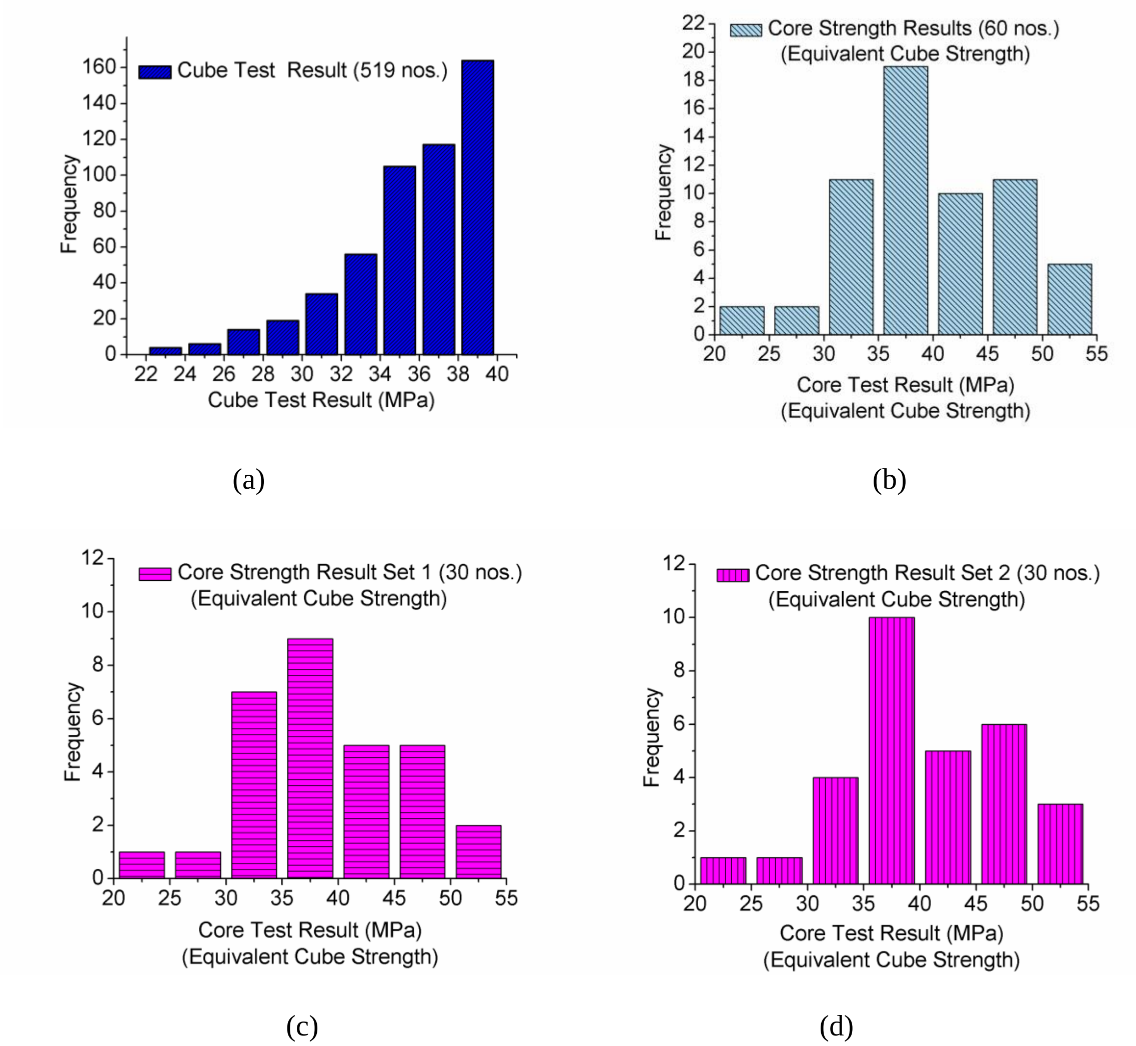


Fig. 1 – Histograms: (a) Original cube results, (b) core results, (c) Core Set 1, (d) Core Set 2

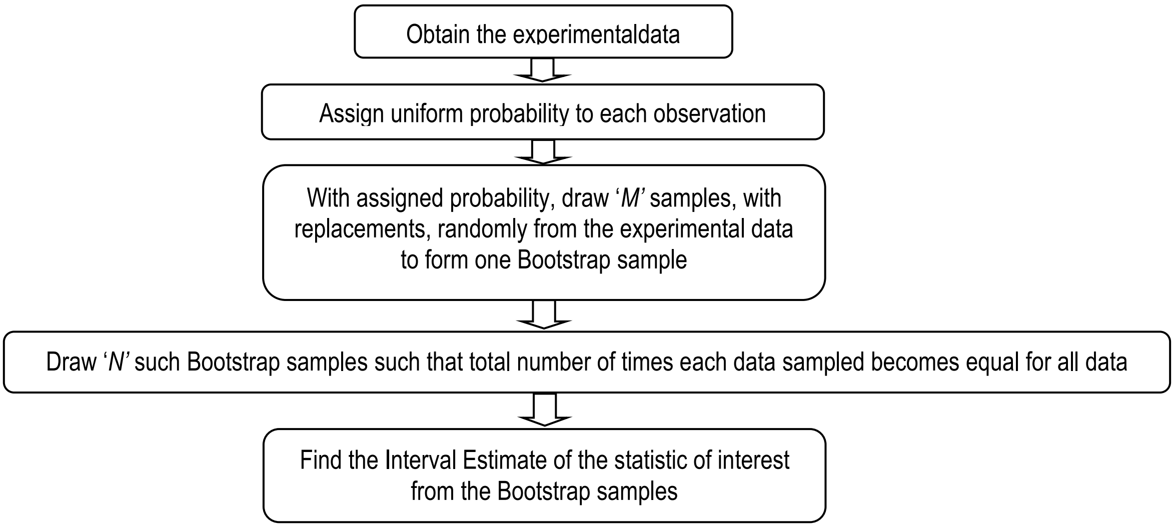


Fig. 2 – Flowchart for balanced non-parametric bootstrap technique

The non-parametric balanced bootstrap method was applied for interval estimation of the statistics of interest, namely, mean, standard deviation, and characteristic strength. The balanced non-parametric bootstrap algorithm is presented in Fig.2.Dealing with sample size of 30, the number of data in each bootstrap sample was varied from 15 to 31 in steps of 2, while keeping the number of bootstrap samples like 200, 400, 600, 800, 1,000, 1,500, 2,000, 3,000, 4,000, and 5,000 and the variations studied. For mean and standard deviation, the variations in the statistics around the recommended [11] value of 2,000 bootstrap samples, and the 25 number of data in each bootstrap sample were studied. Similarly, for characteristic strength, the variations around the recommended [11] 4,000 to 5,000 bootstrap samples and 30 numbers of data in each sample were explored. Similar estimates with Normal approximation theory were thereafter presented with those from bootstrap for comparison purposes. The range of compressive strength, which would contain 90% of the population, was evaluated from the bootstrap and the Normal approximation theory and compared.

For the purpose of seismic re-analysis of the structure for the revised loadings, estimate of the present day concrete strength was required. In this article we intend to evaluate the different statistics of the in situ concrete strength by bootstrap and compare them with those obtained from the Normal approximation theory. Further, the strength obtained from the original cube test results were compared with the in situ strength and inferences would be drawn.

3. Results and discussion

3.1 Estimate by Normal approximation theory: Original cube strength and core strength

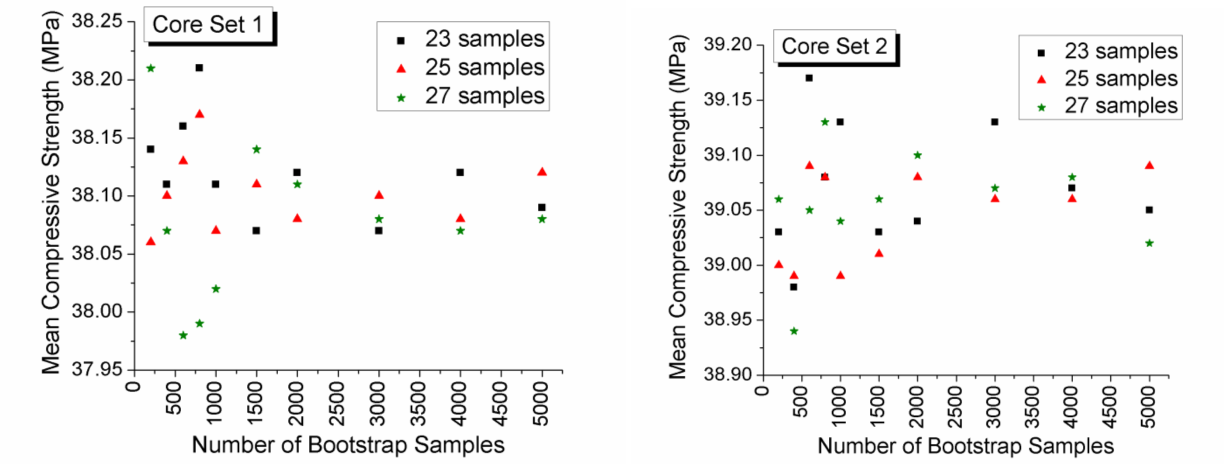
The point estimate of the mean of original concrete cube strength was 35.62 MPa and that of core tests was 39.59 MPa, while the standard deviation was 3.57 MPa for original cubes and 7.27 MPa for cores. Applying Normal approximation theory, the two-sided 90% confidence interval estimated for the mean of the original cubes was 28.28 MPa –39.06 MPa (11.32 MPa) and that core strength was 29.66 MPa –51.32 MPa (21.66 MPa). Thus while the lower limit of the interval increases by around 5%, the upper limit is raised by almost 30% for the core results when compared to the original strength, thereby resulting in almost doubling of the range. According to literature [12], the core results could be less than the corresponding cube test results, which is not observed in this structure. It may be mentioned that the literature [12] reported the strength comparison of the cubes and the cores up to one year from casting. In this case, the comparison is between the cube strength at the time of casting and the core strength around 30 years from casting. Thus, some increase in strength is expected as the concrete gains strength with age due to the progressive hydration of cement and this can explain the increase in the mean and the lower limit. From the closeness of the lower limits of the original cube strength and the core strength, it may be concluded that the present condition of concrete is as good as it was at the time of casting. Further it may be argued that the concrete gone into the structure was as good as the concrete used for casting the cubes, thereby confirming that the quality control was very well implemented at the site. As mentioned earlier, the higher standard deviation in the core results could be due to various factors like the locations of the core in the structure, positions of the core with respect to the lift height, variation in curing, presence of tensile cracks, presence of trapped bleed water, and the directions of the core [12]. The higher spread of the data in the core results contribute to the higher upper limit for 90% interval, and hence the wider interval in the core results.

3.2 Estimate of mean and standard deviation by bootstrap: Variation in number of bootstrap samples

In this section, the authors examine the variation of the strength obtained by bootstrap from the core results when the number of bootstrap samples is varied, as mentioned earlier in the methodology section. The variations obtained for varying the number of bootstrap samples are presented in Fig. 3 for a small variation (± 2) in the recommended number of 25 number of data in each sample.

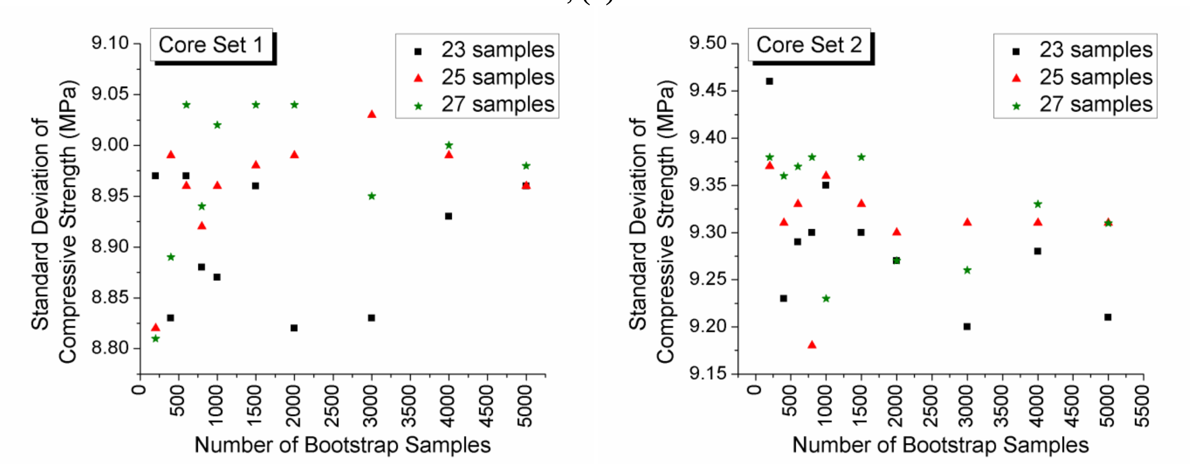
The results are presented for the Core Set 1 and Core Set 2. The mean obtained for number of bootstrap samples more than 1,000 is falling in a narrow band (0.10 MPa) due to different numbers of data in each bootstrap, as can be observed from the figure.

The similar exercise for the standard deviation of the Core Set 1 and Core Set 2 are presented in Fig. 4. In case of standard deviation, it appears that for the recommended number of 1,000 or more, the band in which the data fall is 0.2 MPa in either case. Thus the recommendation obtained from the study of original cube results [11] would be valid for the core tests too. The mean varies between 38 MPa to 39 MPa and the standard deviation between 9.0 MPa and 9.3 MPa for the two core sets, respectively.



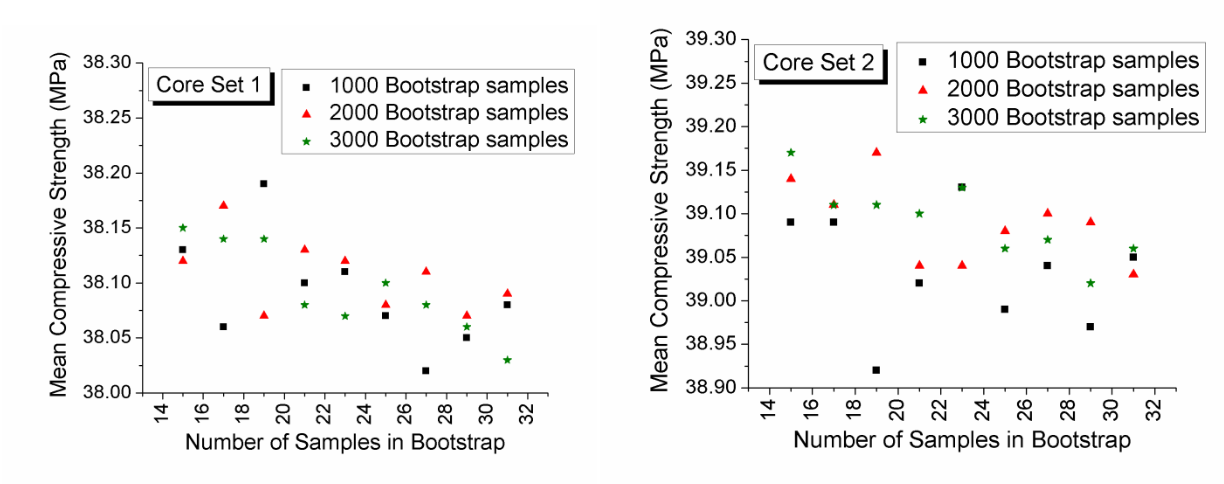
(a) (b)

Fig. 3 – Mean compressive strength of concrete: Variation with different number of bootstrap samples, (a) Core Set 1, (b) Core Set 2



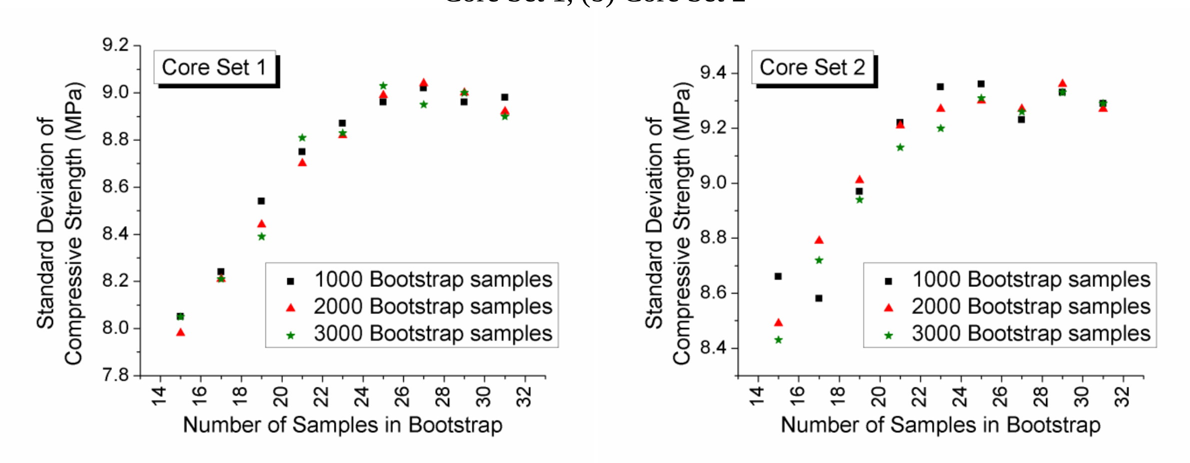
(a) (b)

Fig. 4 – Standard deviation of compressive strength of concrete: Variation with different number of bootstrap samples, (a) Core Set 1, (b) Core Set 2



(a) (b)

Fig. 5 – Mean compressive strength of concrete: Variation with different number of data in each sample, (a) Core Set 1, (b) Core Set 2



(a) (b)

Fig. 6 – Standard deviation of compressive strength of concrete: Variation with different number of data in each sample, (a) Core Set 1, (b) Core Set 2

3.3 Estimate of mean and standard deviation by bootstrap: Variation in number of data in each bootstrap Sample

In this section, the authors examine the variation of the strength obtained by bootstrap from the core results when the number of data in each bootstrap sample is varied, as mentioned earlier in the methodology section. The variations obtained for varying the number of data in each bootstrap sample are presented in Fig. 5 for some variation (±1,000) in the recommended number of 2,000 number of data in each sample.

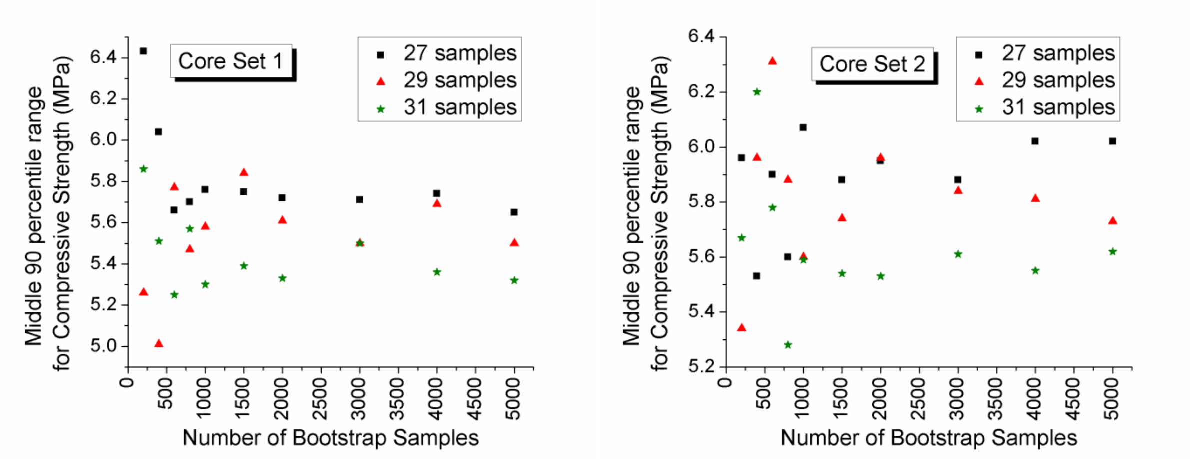
The result is presented for the Core Set 1 and Core Set 2. The mean obtained for number of data in each bootstrap sample more than 1,000 are falling in a narrow band (0.20 MPa) due to different numberbootstrap samples, as can be observed from the figure. The similar exercise for the standard deviation of the Core Set 1 and Core Set 2 is presented in Fig. 6.

In case of standard deviation, it appears that, for the recommended number around 25, the band in which the data fall is 0.2 MPa in either case. Thus, the recommendation obtained from the study of original cube results [11] would be valid for the core tests for both the number of bootstrap samples and the number of data in each bootstrap sample. The mean varies between 38 MPa to 39 MPa and the standard deviation between 9.0 MPa and 9.3 MPa for the two core sets, respectively.

3.4 Estimate of middle 90 percentile range by bootstrap: Variation in number of bootstrap samplesand number of data in each sample

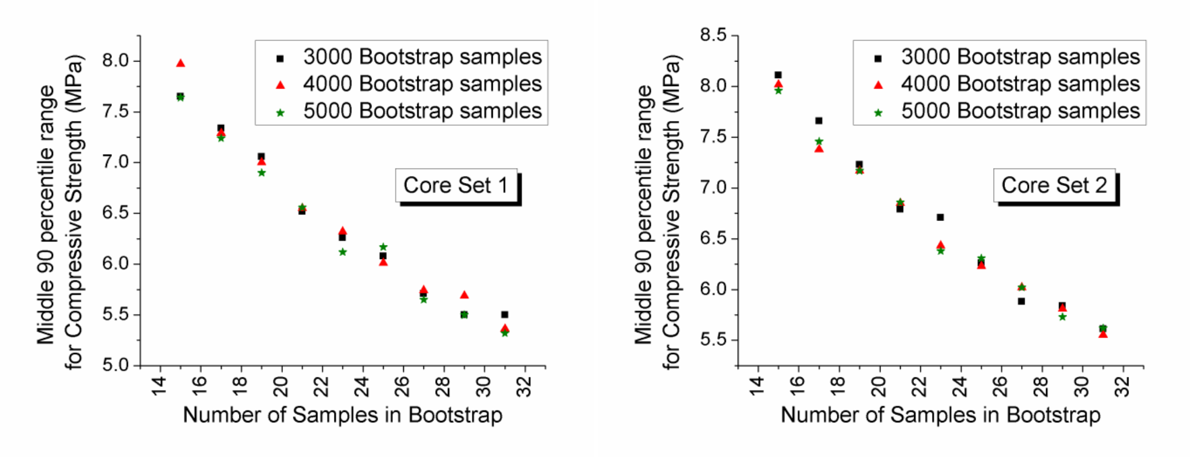
In this section, the middle 90 percentile range for the two subsets from the core results is presented. As before, the variation for different numbers of bootstrap samples is presented in Fig. 7 for a small variation of the recommended value of 30 data in each bootstrap sample. It can be observed that near the recommended number of bootstrap samples equal to 4,000–5,000, the variation is around 10% of the range. In Figure 8 for a small variation of the recommended value of 4,000–5,000 bootstrap samples the variations in the middle 90 percentile range for different number of data in each bootstrap sample is shown.

It can be observed that near the recommended number of bootstrap samples equal to 4,000–5,000, the variation is again around 10% of the range. The range appears to be around 5.32 MPa to 5.62 MPa for the Core Set 1 and Core Set 2 respectively. It may be observed that the range monotonically reduces for higher number of data in each bootstrap sample. This was indicated in literature [11] that, for the extremes, higher number of data in each sample gave better results when employing bootstrap procedure.



(a) (b)

Fig. 7 – Middle 90 percentile range of compressive strength of concrete: Variation with different number of bootstrap samples, (a) Core Set 1, (b) Core Set 2



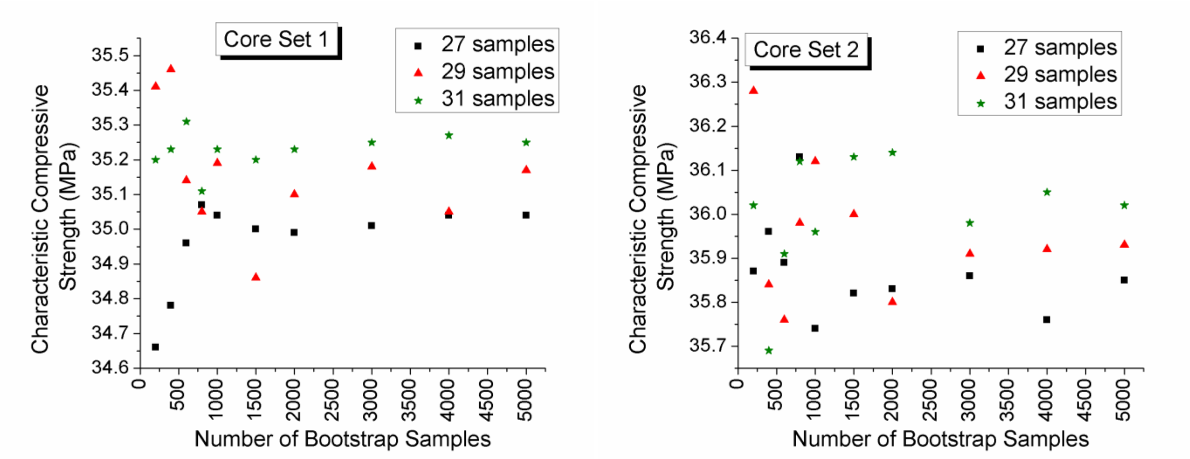
(a) (b)

Fig. 8 – Middle 90 percentile range of compressive strength of concrete: Variation with different number of data in each sample, (a) Core Set 1, (b) Core Set 2

3.5 Estimate of characteristic strength by bootstrap: Variation in number of bootstrap samples and number of data in each sample

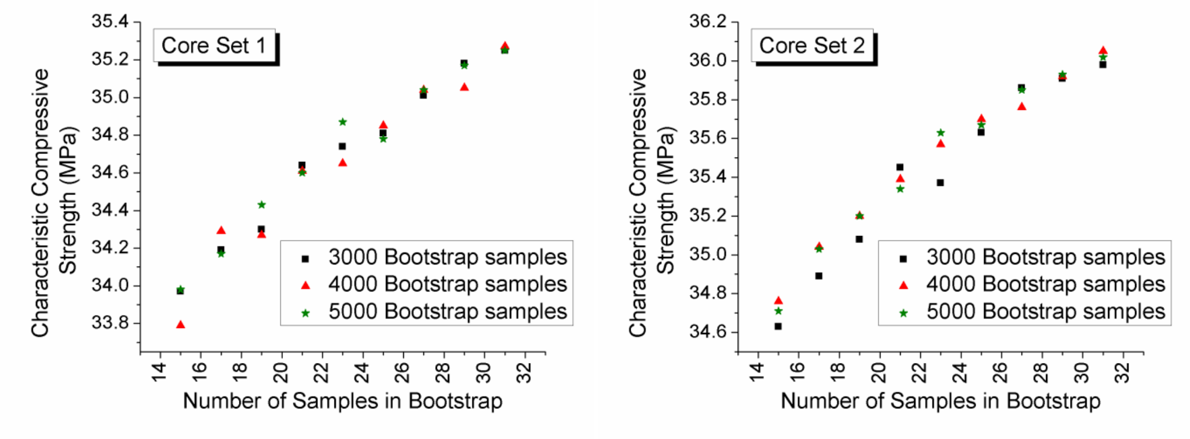
In this section, the characteristic strength according to the Indian standard [13] obtained from the two subsets from the core results is presented. As before, the variation for different numbers of bootstrap samples is presented in Fig. 9 for a small variation of the recommended value of 30 data in each bootstrap sample. It can be observed that near the recommended number of bootstrap samples equal to 4,000–5,000, the variation is less than 1% of the characteristic strength. Subsequently in Fig. 10, for a small variation of the recommended value of 4,000–5,000 bootstrap samples, the variations in the characteristic strength for different number of data in each bootstrap sample are shown.

It can be observed that near the recommended number of data in each bootstrap samples equal to 30, the variation is again less than 1% of the compressive strength. The characteristic strength obtained from the Core Set 1 and Core Set 2 by applying bootstrap is 35.25 MPa and 36.02 MPa, respectively. Similar to the middle 90 percentile range, it may be observed that the characteristic strength monotonically increases for higher number of data in each bootstrap sample. This was indicated in literature [11], that for the extremes, higher number of data in each sample gave better results when employing bootstrap procedure.



(a) (b)

Fig. 9 – Characteristic Strength of Concrete: Variation with different number of bootstrap samples, (a) Core Set 1, (b) Core Set 2



(a) (b)

Fig. 10 – Characteristic Strength of Concrete: Variation with different number of data in each sample, (a) Core Set 1, (b) Core Set 2

3.6 Discussion on the different statistics of concrete core results by Normal approximation theory and bootstrap

The estimates of the various statistics of compressive strength of concrete from the core results (Core Set 1 and Core Set 2) by Normal approximation theory and bootstrap (the recommended numbers of bootstrap samples and number of data in each sample according to literature [11] are presented in Table 2. It is observed that with both the Core Set 1 and the Core Set 2, the mean obtained by bootstrap is lower than that by Normal approximation by around 1 MPa. As was indicated in literature [11], the standard deviation by the Normal theory is lower than that by bootstrap by around 1.7–1.8 MPa. Thus, as before, the bootstrap estimate of mean and standard deviation turns out to be conservative when compared to the estimates by Normal approximation. It was further observed that with higher number of bootstrap samples or with higher number of data in each bootstrap sample, the estimates are less affected by choice of the number of bootstrap samples or number of data in each sample. This is in similar line as the earlier study by the authors [11].

In bootstrap estimate of middle 90 percentile range or the characteristic strength of the core results, it was observed that with higher number of bootstrap samples the estimates are less affected by choice of the corresponding number of bootstrap samples. Similarly, for higher number of data in each bootstrap sample the estimates are less affected by choice of the corresponding number of data in each sample. It may be further mentioned that the characteristic strength as estimated from the bootstrap increases with increasing number of data in each bootstrap sample. This behavior is also similar to the earlier study [11]. In the non-parametric estimate of the middle 90 percentile range by bootstrap, it is seen to be almost 25% of the corresponding Normal approximation estimate, indicating that the central tendency of the data would be much higher than those predicted by the assumption of normality. Similarly, the characteristic strength evaluated by the bootstrap is around 20% more than those evaluated by Normal approximation in both core datasets. This finding is in contrast to the observations in the last study by the authors [11], where the estimate from Normal approximation was consistently higher.

Intuitively, it would appear that the higher standard deviation and the lower middle 90 percentile range as estimated in the bootstrap is contradictory. But that intuition is based on the general impression of the normal distribution of the data and parametric estimation of percentiles from the assumed distribution and the evaluated parameters of the assumed distribution. Assumption of Normal distribution imposes certain characteristics to the dataset investigated and thereby on the derived quantities like percentiles [17]. In the non-parametric bootstrap procedure, the concrete properties are derived directly from the bootstrap samples generated and does not depend on any assumed distribution or estimated distribution parameters [4]. This is the strength of the non-parametric approach that the estimates reflect the true characteristics of the dataset, and is unencumbered by the assumptions that are mandatory for the parametric methods [4]. Such variations for estimated percentiles from normal approximation theory and from bootstrap sampling have been mentioned in literature [4]. For limited datasets where the assumption of normality is not always established, the bootstrap estimate would be suitable to have a better representation of the data and better estimates of the different percentiles directly from the datasets, without any assumed distribution and the evaluated properties of the assumed distribution.

Table 2 – Estimates of the statistics of compressive strength of concrete from core test results

Statistic	Core Set 1 (30 nos.)		Core Set 2 (30 nos.)	
	Normal approx. theory	Bootstrap method	Normal approx. theory	Bootstrap method
Mean (MPa)	39.09	38.03	40.09	39.06
Standard Deviation (MPa)	7.18	8.90	7.45	9.29
Middle 90 Percentile Range (MPa)	20.82	5.32	22.32	5.62
Characteristic Strength according to Indian Standard [13] (MPa)	29.62	35.25	29.97	36.02

3.7 Discussion on concrete strength by bootstrap and Normal approximation theory: Original cube strength and core strength

In this section the concrete strength from the original cube test results and the present core test results are compared for both the Normal Approximation Theory (parametric) and the bootstrap technique (non-parametric) and presented in Table 3 and Table 4, respectively. As mentioned earlier for Normal approximation theory (Table 3), there is increase in both the mean and standard deviation of compressive strength when evaluated from the core results, by 11% and 103%, respectively, while the characteristic strength (lower 5 percentile value) reduces by 7%. In the estimates by bootstrap (Table 4), there is increase in both the mean and standard deviation of compressive strength when evaluated from the core results, by 8.6% and 156%, respectively, while the characteristic strength (lower 5 percentile value) increases by 3.6%.

It is noteworthy that contrary to the suggestions in some literature [12], experimental studies had reported higher strength for the cores as compared to the cubes for normal strength concrete [16]. In the present study too, the mean of the equivalent cube compressive strength is higher for the core tests when compared to the original cube results. Some increase in strength is expected as the concrete gains strength with age due to the progressive hydration of cement and this can explain the increase in the mean. It may be inferred that the present condition of concrete is as good as it was at the time of casting. Further it may be argued that the concrete gone into the structure was as good as the concrete used for casting the cubes, thereby confirming that the quality control was very well implemented at the site.

Table 3 – Estimates of the concrete strength by normal approximation theory from original cube and core test results

Concrete strength	Original cube result [11] (MPa)	Core result (MPa)
Mean	35.62	39.59
Standard deviation	3.57	7.27
Characteristic strength	29.73	27.59

NOTE: Characteristic strength by Indian Standard [13]

Table 4 – Estimates of the concrete strength by bootstrap from original cube and core test results

Concrete strength	Original cube result [11] (MPa)	Core result (MPa)
Mean	35.50	38.55
Standard Deviation	3.55	9.10
Characteristic strength	34.4	35.64

NOTE: Characteristic strength by Indian Standard [13]

There has been an increase of more than 100% in the standard deviation of the compressive strength of concrete in case of core results for both methods. The cubes are cast, compacted, cured and tested under uniform condition and thus the standard deviation of the cube strength is expected to be low, provided the quality control was implemented properly. In case of the cores coming from the different structural elements, the various factors such as compaction, height of lift, curing, and sampling are non-uniform. Further, the cores came from different parts of the structure that had undergone different deformations due to the loads on the structure. As mentioned earlier, the higher standard deviation in the core results could be due to various factors such as the locations of the core in the structure, positions of the core with respect to the lift height, variation in curing, presence of tensile cracks, presence of trapped bleed water, and the directions of the core [12]. In case of the characteristic strength estimation, though, by Normal approximation, the value reduces marginally in case of cores, with bootstrap there is marginal increase. In general, it is concluded that the concrete quality as existing in present day on the structure is as good as the concrete quality of the cubes taken during the time of casting of the structure more than 25 years back. This could be attributed to the good quality control during casting and periodic maintenance for preserving the quality as was performed for the structure under examination.

4. Summary and conclusions

In this article, equivalent cube strength of core test results obtained from a structure had been examined with non-parametric bootstrap for the purpose of estimation of the various statistics and characteristic strength of concrete. The estimates from the Normal approximation theory were also presented for comparison purposes. For applying the bootstrap technique, the recommendation in literature [11] regarding the number of bootstrap samples and the number of data in each sample were utilized and the result of small variations around those numbers was studied. Thereafter, the estimates obtained from the original cube test results as reported in literature [11] were compared to those obtained from the core test results.

The balanced non-parametric bootstrap technique was observed to be robust to the choice of initial sample as consistent results were obtained from the two representative sets of data from the core results. The recommended numbers of 2,000 bootstrap samples with 25 data in each sample for estimation of the mean and standard deviation, and 4,000-5,000 bootstrap samples with 30 data in each sample for estimation of the extreme percentiles as reported for cube results in literature [11] were found to be equally applicable for the core results. Similar to the earlier study [11] with cube results, higher number of bootstrap samples and higher number of data in each sample gave better estimates for core results. The mean and standard deviation estimated by the bootstrap were found to be conservative as compared to Normal approximation theory.

However, contrary to the earlier study [11], the characteristic strength was found to be higher in bootstrap method when compared to the Normal approximation. The possible reason for this behavior could be that in non-parametric estimate like bootstrap, the desired percentile is estimated directly from the dataset, and not estimated from the estimated properties of the assumed distribution, as is performed in Normal approximation theory. For limited datasets where the assumption of normality is not always established, the bootstrap estimate would be suitable to have a better representation of the data and better estimates of the different percentiles directly from the datasets, without any assumed distribution and the estimated properties of the assumed distribution.

The comparison of the estimates from cube test results and the core results by both Normal approximation as well as bootstrap indicated that the mean was marginally higher in the core results while the standard deviation was more than 100% higher. Some increase in strength is expected as the concrete gains strength with age due to the progressive hydration of cement and this can explain the increase in the mean. The higher standard deviation in the core results could be due to various factors like the locations of the core in the structure, positions of the core with respect to the lift height, variation in curing, presence of tensile cracks, presence of trapped bleed water, and the directions of the core [12]. The higher strength for the cores as compared to the cubes had been reported in literature [16] for normal strength concrete, as in the present case.

The estimated characteristic strength from the cubes and the cores varied marginally in either method of estimation. Hence, it is concluded that the concrete quality as existing in present day on the structure is as good as the concrete quality of the cubes taken during the time of casting of the structure more than 25 years back. This could be attributed to the good quality control during casting and periodic maintenance for preserving the quality as was performed for the structure under examination. It is important to note that the inferences drawn in this article did not account for the relative accuracy of the data used for the study, namely, the cube test results and the core test results.

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